Stateful Manifest Contracts

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Abstract

This paper studies hybrid contract verification for an imperative higher-order language based on a so-called *manifest contract system*. In manifest contract systems, contracts are part of static types and contract verification is hybrid in the sense that some contracts are statically verified, typically by subtyping, but others are dynamically by casts. It is, however, not trivial to extend existing manifest contract systems, which have been designed mostly for pure functional languages, to imperative features, mainly because of the lack of flow-sensitivity, which should be taken into account in verifying imperative programs statically.

We develop an imperative higher-order manifest contract system $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ for flow-sensitive hybrid contract verification. We introduce a *computational* variant of Nanevski et al's Hoare types, which are flow-sensitive types to represent pre- and postconditions of impure computation. Our Hoare types are computational in the sense that pre- and postconditions are given by Booleans in the same language as programs so that they are dynamically verifiable. $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ also supports refinement types as in existing manifest contract systems to describe flow-insensitive, state-independent contracts of pure computation. While it is desirable that any—possibly state-manipulating—predicate can be used in contracts, abuse of stateful operations will break the system. To control stateful operations in contracts, we introduce a region-based effect system, which allows contracts in refinement types and computational Hoare types to manipulate states, as long as they are *observationally* pure and read-only, respectively. We show that dynamic contract checking in our calculus is consistent with static typing in the sense that the final result obtained without dynamic contract violations satisfies contracts in its static type. It in particular means that the state after stateful computations satisfies their postconditions.

As in some of prior manifest contract systems, static contract verification in this work is "post facto," that is, we first define our manifest contract system so that all contracts are checked at run time, formalize conditions when dynamic checks can be removed safely, and show that programs with and without such removable checks are contextually equivalent. We also apply the idea of post facto verification to *region-based* local reasoning, inspired by the frame rule of Separation Logic.

Categories and Subject Descriptors D.2.4 [*Software Engineering*]: Software/Program Verification—Programming by contracts; D.3.1 [*Programming Languages*]: Formal Definitions and Theory; F.3.1 [*Specifying and Verifying and Reasoning about Programs*]: [Assertions]

General Terms Languages, Design, Theory, Verification

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1. Introduction

1.1 Hybrid Contract Verification for Imperative Languages

Since "Design by Contracts" by Meyer [33], *software contracts* have been a broadly used, prominent tool to describe program specifications and construct robust software. The fact that contracts are executable—that is, they are written in the same language as programs—distinguishes them from other specification languages and makes it possible to verify programs either dynamically or statically. While dynamic verification is the original purpose of contracts and much work along this has been studied [9, 13, 14, 16, 17, 25, 33, 54, 58, 65], static verification with contracts is also drawing attention—for example, there is recent work to integrate contracts with dependent types [5, 46, 62, 63] and symbolic execution [40, 59, 72]. Dynamic and static approaches to contract verification have complementary pros and cons: dynamic verification is easy to use but it makes program execution inefficient and may miss critical errors; and static verification can check programs exhaustively and do not affect run-time behavior of programs but it often imposes heavy burden on programmers and restricts programming features which can be used in programs and/or contracts.

To take advantages of both static and dynamic verification, *hybrid contract verification* has been studied [3, 18, 19, 23, 30, 42, 71]. In hybrid verification, the two verification mechanisms are integrated so that as many contracts as possible are verified statically and others are dynamically. Hybrid verification helps reasonable, certified software development. One possible scenario to utilize it is to verify critical parts of a program statically by rich contracts and check that program points to enter the critical parts satisfies the contracts at run time.

Although hybrid verification has a potential to realize rapid development of reliable software, it has been so far studied mostly for pure languages. However, practical software is usually written in an imperative language and so it is necessary to equip hybrid verification with imperative features for applying it to practical software development. In fact, there is little work on hybrid contract verification with states [19, 42], but it is insufficient for verifying

* A part of the work has been done while he was at Kyoto University.

```
type tbl
val mem : tbl -> string -> bool
val add : t:tbl ->
          s:{s:string|not (mem t s)} ->
          {u:unit|mem t s}
val fresh_str : t:tbl ->
                {s:string|not (mem t s)}

let t : tbl = ...  (* make a table *)
let s : {s:string|not (mem t s)} = fresh_str t

add t s; add t s
```

Figure 1. A problematic program breaking value inversion and the invariant on string tables.

stateful programs because of the lack of support for *state-dependent contracts*, which are important in verifying, say, abstract mutable data structures such as hash tables.

The goal of this work is to advance hybrid contract verification so that stateful programs with state-dependent contracts can be verified. In this paper, we design hybrid verification based on *manifest contract systems* [3, 18, 22, 30, 48, 49], which are a theoretical model to integrate static and dynamic contract verification. Manifest contract systems are adequate for studying hybrid contract verification in that they do not depend on specific static verification technologies and so we can adopt any desirable static approaches in implementing them as languages. In what follows, after describing manifest contract systems briefly, we show challenges in extending manifest contract systems with imperative features and how we address them.

1.2 Manifest Contracts for Imperative Languages

Manifest contract systems are a theoretical model of intermediate languages for hybrid contract verification. In them, contracts are embedded into static types, in particular, refinement types of the form $\{x{:}T \mid e\}$, which means values v (of type T) satisfying the Boolean expression e, i.e., $[\,v/x\,]\,e$ reduces to true. Contract e, also called a refinement, can use any code fragments; for example, a refinement type of positive integers can be represented by $\{x{:}\mathsf{int} \mid x > 0\}$ and that of prime numbers can be by $\{x{:}\mathsf{int} \mid \mathtt{prime?}\,x\}$ if Boolean program function `prime?`, which returns whether the argument is a prime number, is given. Contract verification in existing manifest contract systems is formalized as a kind of type conversion: static verification is reduced to subtyping checking of refinement types and dynamic verification is performed by *casts* $\langle T_1 \Leftarrow T_2 \rangle^{\ell}$, which converts inhabitants of source type T_2 to target type T_1 at run time and, if the conversion fails, an uncatchable exception will be raised. To connect static and dynamic verification, manifest contract systems are equipped with a key property called *value inversion*, which states that a value of $\{x{:}T \mid e\}$ must satisfy contract e.

Existing manifest contract systems have been studied mostly for pure languages and it is not trivial to extend them to imperative features, mainly because of the lack of flow-sensitivity, which has to be taken into account in verifying stateful programs with state-dependent contracts statically. Although naive introduction of states to pure manifest contract systems makes it possible to embed state-dependent contracts into refinement types, it is unreasonable because refinement types are flow-insensitive, in the sense that their inhabitants are supposed to satisfy their contracts everywhere, but state-dependent contracts can be invalidated by assignment operations; as a result, value inversion would be broken. To clarify what would happen in the naive extension, let us consider string tables, which are implemented by mutable references to string lists whose elements are pairwise distinct. Figure 1 shows an ML-like program where state-dependent contracts for functions of string tables are embedded into refinement types. The program provides three functions: `mem`, which returns whether a given table contains a given string, `add`, which adds a given new string to a given table, and `fresh_str`, which returns some string not contained in a given table. To preserve the invariant on string tables, a contract for `add` requires a given string not to be a member of a given string table. The return type of `fresh_str` refers to a given table and means that it produces a fresh string for the table. Notice that `add` and `fresh_str` are given dependent function types—in a dependent function type `x:T1->T2`, return type `T2` can refer to the argument of type `T1` as `x`. Given table `t` and string `s` produced by `fresh_str` with `t`, flow-insensitivity of refinement types allows us to refer to `s` as strings satisfying `not (mem t s)` everywhere. Since the type of `s` coincides with the argument to `add t`, we can pass `s` to `add t` in both calls on the last line. After the first call to `add`, table `t` should include string `s`, while the type of `s` still claims that `t` does not include `s`—value inversion is broken! Furthermore, violation of value inversion would cause manifest contract systems to accept erroneous programs as Figure 1, where the second call to `add` will break the invariant on string tables.

A lesson from the discussion above is that, though state-dependent contracts are necessary for specifying the behavior of stateful programs, they may be invalidated during program execution. Although indeed there is extensive work which studies contracts for stateful programs [13, 14, 19, 24, 55], their work is not very satisfactory because they (1) focus on how to enforce contracts for references but not on dealing with state-dependent contracts, or (2) restrict contracts excessively. For example, Flanagan, Freund, and Tomb [19] have addressed the problem by allowing only pure expressions as refinements, but contracts for operations on mutable data structures such as `add` and `fresh_str` could not be given because they are usually state-dependent.

1.3 Our Work

In this paper, we develop an imperative higher-order manifest contract system $\lambda^{\mathsf{H}}_{\mathsf{ref}}$ with a flow-sensitive type system for state-dependent contracts. In addition to refinement types to describe state-independent contracts, we introduce *computational Hoare types*, a variant of Hoare types in Nanevski et al.'s Hoare Type Theory (HTT) [37, 38], to describe state-dependent contracts. For example, contracts for `add` and `fresh_str` are expressed by computational Hoare types since they are state-dependent, while contracts for positive numbers and prime numbers are expressed by refinement types since they are state-independent. Our Hoare types are computational in the sense that, unlike HTT, predicates are contracts, that is, executable computations so that they are dynamically verifiable. (In this paper, computational Hoare types are simply called Hoare types if it is clear from the context that the computational version is meant.) The type system is flow-sensitive—that is, it tracks what state-dependent contracts hold and what may have been invalidated after imperative operations—with the help of Hoare types. Flow-sensitivity of the type system prevents the problem in Section 1.2 by finding out that the contract "`not (mem t s)`" may be invalidated by the assignment to table `t` in the first call to `add`.

We define $\lambda^{\mathsf{H}}_{\mathsf{ref}}$ by following Belo et al. [3], who have proposed a general approach to defining manifest contract calculi with "post facto" static verification. Their approach sets subtyping apart from the type system of a calculus so that all contract checks are made explicit as casts; static verification amounts to detecting and eliminating necessarily successful casts by examining their source and target types before executing programs. Such an approach allows us to separate issues in designing a mechanism for dynamic checking

and methods for static verification, making metatheory of the calculus significantly less complicated (if not simple). We will show that dynamic contract checking in $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ is consistent with static typing by establishing two properties (as well as usual progress and preservation [44, 67]): one is value inversion mentioned above and the other is that the final state after stateful computations satisfies their postconditions. Then, we study, as a basis for static verification, sufficient conditions for dynamic check of state-dependent contracts to be eliminated without changing the semantics of a program.

What follows describes our contributions in more detail.

Hoare types with state-dependent contracts Hoare types are a kind of dependent types inspired by Hoare triples in Hoare logic [26]. A Hoare triple $\{P\}\, c\, \{Q\}$ means that stateful computation c demands that precondition P hold and, after computing c, guarantees that postcondition Q holds. Similarly, Hoare types, written as $\{A_1\}x{:}T\{A_2\}$ in this work, denote stateful computations which assume precondition A_1 before their execution, return values of T, and ensure postcondition A_2 after their execution; variable x represents the return values of the computations in A_2. A_1 and A_2 are possibly *state-dependent* contracts and program states have to satisfy these contracts before and after executing computations of this Hoare type. From a denotational point of view, computations of the Hoare type can be interpreted as functions over states such that they take states satisfying A_1 and return states satisfying A_2 with some value x of T. Using Hoare types, for example, the contract for `add` can be presented as:

$$t{:}\mathsf{tbl} \to s{:}\mathsf{string} \to \{\mathtt{not}\,(\mathtt{mem}\; t\; s)\}x{:}\mathsf{unit}\{\mathtt{mem}\; t\; s\},$$

which describes that `add` accepts a string table t and a string s and insists that s is not a member of t in the state in calling `add`; when these all preconditions are met, `add` computes something to result in the state where t contains s. Following HTT, $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ classifies programs into pure and impure fragments in the monadic style [35, 36, 64], and contracts for pure computations are represented by refinement types as in prior manifest calculi and ones for impure computations are by Hoare types.

Dynamic checking All contracts in $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ are checked at run time by using either of two mechanisms: *casts* (called also dynamic type conversion), a customary means in manifest contracts to check refinements in pure fragments, and *assertions*, a new means to check pre- and postconditions of stateful computations—in both mechanisms, if a dynamic check fails, an uncatchable exception (called *blame* [16]) will be raised to notify contract violation. We extend the cast-based mechanism to reference types and Hoare types—casts for these types produce wrappers which check properties about states when they evaluate in impure fragments. For reference types, we use the idea of views and guards in the earlier work [13, 19, 24, 55]. $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ also provides an assertion-based checking mechanism to check pre- and postconditions at run time. When computation c needs some preconditions to be satisfied, we can check them at run time before executing c; if they hold, the remaining computation proceeds; otherwise, an uncatchable exception is raised. On the contrary, when c is required to ensure postconditions, we can also check them at run time with the execution result of c; if they hold, the whole computation returns the result of c; otherwise, an uncatchable exception is raised.

Type-and-effect system We design a type system of $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ to not only ensure that values of refinement types satisfy their refinements, which is a key property in manifest contracts [21, 48, 49], but also identify how long state-dependent contracts that have been checked remain true. We hope that contracts are as expressive as possible, but computations with assignments to references in programs should not be accepted as contracts because, intuitively, contracts are specifications and so should not affect the program behavior and, technically, type soundness needs that run-time contract checking be recalculatable, at least immediately after ending the check, but such assignments would make it impossible. To control stateful operations in contracts, we introduce a region-based effect system [8, 31, 60], which allows contracts in refinement types and Hoare types to manipulate references as long as they are *observationally* pure and read-only, respectively—that is, refinements can manipulate only references allocated in regions local to themselves and pre- and postconditions can dereference any memory cell in addition to arbitrary manipulation of locally allocated references. The effect system ensures that those local references never escape to programs. In this paper, we simply call observationally pure and observationally read-only contracts pure and read-only, respectively.

Static verification of state-dependent contracts The effect system is also useful for static verification of state-dependent contracts. Following Belo et al. [3], this work studies "post facto" static verification, that is, we define $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ so that all contracts are checked at run time, formalize what dynamic checks can be eliminated safely, and show that programs with and without such checks are contextually equivalent. Intuitively, dynamic checks of contracts can be eliminated when their satisfaction is derived from other, already established contracts. For example, computations ensuring postcondition that table t contains some string also guarantee another postcondition that the size of table t is larger than 0 without additional assertions *if it is proven that the former implies the latter*—although what it says would seem trivial, showing soundness of static verification in manifest contracts is usually not easy [3, 30, 49].

We also apply post facto verification to *region-based* local reasoning, which is inspired by the frame rule in Separation Logic [41, 45] and means that satisfaction of preconditions which do not refer to references mutated by a computation is preserved even after executing the computation. The local reasoning would enable modular verification because we can verify subcomponents of a program without having to know the entire contract of the program. For example, thanks to this local reasoning, $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ can accept program "`let s = fresh_str t; let b = mem t "foo"; add t s`" without inserting run-time checks even if the postcondition of `mem` does not state explicitly that `s` is not contained in `t` still.

Although, unlike the original work of manifest contracts [3, 18, 30], this work does not study static verification of refinements due to the difficulty from higher-order types, we expect that our work would be a foundation for such verification.

Organization The rest of this paper is organized as follows: we describe an overview of $\lambda^{\mathrm{H}}_{\mathsf{ref}}$ in Section 2; Sections 3, 4, and 5 offer the program syntax, the type system, and the operational semantics of $\lambda^{\mathrm{H}}_{\mathsf{ref}}$, respectively, and Section 6 shows type soundness after extending the type system with run-time typing rules. After showing static verification in Section 7, we discuss related work in Section 8 and conclude in Section 9.

We omit the proofs of our technical development from this paper; interested readers can refer to the full version at: `http://www.fos.kuis.kyoto-u.ac.jp/~t-sekiym/papers/refh/refh_full.pdf`

2. Overview of $\lambda^{\mathbf{H}}_{\mathsf{ref}}$

This section gives an overview of $\lambda^{\mathrm{H}}_{\mathsf{ref}}$, focusing on four key ideas: first, classification of programs to pure and impure fragments in the monadic style; second, computational Hoare types; third, a region-based effect system; and finally, an assertion-based mechanism to check pre- and postconditions of Hoare types at run time.

2.1 Terms and Computations in Monadic Style

Following HTT, the program syntax of our calculus consists of two syntax classes: terms, which are considered as "almost" pure program fragments and can be typed at refinement types, and computations, which are considered as impure program fragments and can be typed at Hoare types. Terms do not manipulate states but include cast applications, so they raise exceptions if some cast fails; this is the reason why terms are "almost" pure. To classify programs into terms and computations, we adopt the monadic style [35, 36, 64], which is a well-known syntactic discipline to distinguish between program fragments with and without computational effects.

In fact, we would meet difficulties without any mechanism to distinguish pure and impure computation. One issue is how to deal with dependent types. Let us consider function application $e_1\, e_2$ where e_1 and e_2 are typed at dependent function type $x{:}T_1 \to T_2$ and type T_1. We expect the result type of $e_1\, e_2$ to be return type T_2, but T_2 is dependent on the argument variable x of T_1. Thus, a standard typing rule [53] gives $e_1\, e_2$ type $[\, e_2/x \,]\, T_2$ with substitution of e_2 for x. However, this approach is unsound if term e_2 involves stateful effects. This problem has been addressed by using existential quantifier to hide information of substituted terms [15, 29]; however, this remedy cannot be directly applied to our calculus—hidden information cannot be recovered, unlike Knowles and Flanagan [29], due to effects in terms, and, even worse, it is very difficult (possibly impossible) to check specifications with existential quantifier at run time in an algorithmic way.

The monadic style solves this problem. Since arguments to functions must be pure, we can adopt the standard substitution-based rule. It is also convenient in formalizing typing rules, thanks to the fact that intermediate results during computation are all named. Another advantage is that it is easy to analyze when state-dependent contracts may be invalidated since the monadic style sequentializes effects, in particular, assignment, which is the only effect that can invalidate them.

Terms Terms e are mostly from the lambda calculus and manifest contracts. The most distinguishing construct from manifest contracts is casts $\langle T_1 \Leftarrow T_2 \rangle^{\ell}$, which check that, when applied to values of source type T_2, they can behave as target type T_1. For example, since 3 is positive, the cast application $\langle \{x{:}\mathsf{int} \mid x > 0\} \Leftarrow \mathsf{int} \rangle^{\ell}\, 3$ succeeds and returns 3 whereas, since 0 is not positive, $\langle \{x{:}\mathsf{int} \mid x > 0\} \Leftarrow \mathsf{int} \rangle^{\ell}\, 0$ gives rise to an uncatchable exception $\Uparrow\ell$ with label ℓ to notify contract violation (the label ℓ is used to identify the program point with the cast). Terms also contain do-expressions, a usual construct in monadic languages. do-expressions, written as $\mathsf{do}\, c$ (where c is a computation discussed below), are suspended computations and are introduced to deal with computations in the context of terms. Using do-expressions, for example, we can write higher-order stateful functions (that is, functions taking and/or returning stateful computations) and implement recursive functions via Landin's knot. do-expressions will be executed when they are connected with the top-level computation.

Computations Computations, denoted by c, are constructed as usual by two constructs: return and bind. The return construct $\mathsf{return}\, e$ returns the evaluation result of term e as a computation result. The bind construct $x \leftarrow e_1; c_2$ evaluates term e_1 to do-expression $\mathsf{do}\, c_1$, and then computes c_1 and c_2 sequentially. The computation c_2 can refer to the result of c_1 as x. Moreover, since our interest is in stateful programs, computations can deal with operations on mutable references. Such operations are written as

$$x \Leftarrow \mathsf{ref}\, e_1; c_2 \qquad x \Leftarrow !e_1; c_2 \qquad x \Leftarrow e_1 := e_1'; c_2$$

which represent memory allocation, dereference, and assignment, respectively. After doing the corresponding action, they compute c_2 by binding x to the result of the action. The assignment action returns the unit value.

Example Let us consider a function that takes a reference that points to an integer value, increments the contents, and then returns the old value of the reference. Such a function can be written as:

$$f \stackrel{\text{def}}{=} \lambda x{:}\mathsf{Ref\ int}.\mathsf{do}\ y \Leftarrow !x; \_ \Leftarrow x := y + 1; \mathsf{return}\ y$$

Here, $\mathsf{Ref\ int}$ is the type of integer references and "_" means an unused variable. We can call f by passing a reference to an integer:

$$x \Leftarrow \mathsf{ref}\ 1; y \leftarrow f\, x; z \Leftarrow !x; \mathsf{return}\ y + z$$

This computation returns integer 3 because the contents of x are updated to 2 and f returns the old value 1.

2.2 Hoare Types

Hoare types have pre- and postconditions of stateful computations. In our work, these conditions are sequences A of Boolean computations. Intuitively, condition A means the conjunction of all contracts in A; since the empty sequence means the contract which always hold, we write $\top$ for it. Formalizing pre- and postconditions as sequences of computations simplifies the metatheory of our calculus—e.g., it allows strengthening preconditions and weakening postconditions in a natural way. Computations of Hoare type $\{A_1\}x{:}T\{A_2\}$ demand A_1, produces values v of type T (if any), and ensures $[\, v/x \,]\, A_2$.

Hoare types are flow-sensitive in the sense that, if computation c_2 will be executed immediately after computation c_1, the precondition of the Hoare type of c_2 has to syntactically coincide with the postcondition of the Hoare type of c_1. If they do not, run-time checks by assertion to confirm that the precondition of c_2 holds are necessary immediately before executing c_2. Fortunately, as will be discussed in Section 7, if the precondition of c_2 is *implied* from the postcondition of c_1, we can eliminate such run-time checks "post facto."

Example Let us consider a contract that a computation requires reference x to point to an integer list each of which is a prime number, add new prime numbers to x, and returns the length of the result list. Using Hoare types, the contract is given as follows:

$$\begin{array}{l}\{y \Leftarrow !x; \mathsf{return}\ (\texttt{for\_all prime}?\ y)\} \\ \quad z{:}\mathsf{int} \\ \{y \Leftarrow !x; \mathsf{return}\ (\texttt{for\_all prime}?\ y)\ \&\ (\texttt{length}\ y = z)\}\end{array}$$

where $\texttt{for\_all prime}?\ y$ returns whether each element in integer list y is a prime number and `length` returns the length of the argument list. The pre- and postcondition are allowed to read an integer list from reference x since contracts of Hoare types can be state-dependent, unlike refinement types. Similarly, `mem`, `fresh_str`, and `add` in Figure 1 can be given:

$$\begin{array}{ll}\texttt{mem} & : \mathsf{tbl} \to \mathsf{string} \to \{\top\}x{:}\mathsf{bool}\{\top\} \\ \texttt{fresh\_str} & : t{:}\mathsf{tbl} \to \{\top\}s{:}\mathsf{string}\{y \leftarrow \texttt{mem}\ t\ s; \mathsf{return}\ \texttt{not}\ y\} \\ \texttt{add} & : t{:}\mathsf{tbl} \to s{:}\mathsf{string} \to \{y \leftarrow \texttt{mem}\ t\ s; \mathsf{return}\ \texttt{not}\ y\} \\ & \qquad\qquad x{:}\mathsf{unit} \\ & \qquad\qquad \{y \leftarrow \texttt{mem}\ t\ s; \mathsf{return}\ y\}\end{array}$$

Thanks to flow-sensitivity of Hoare types, while a program which adds a string produced by `fresh_str` to a string table once can be accepted:

$$\lambda t{:}\mathsf{tbl}.\mathsf{do}\ s \leftarrow \texttt{fresh\_str}\ t; \_ \leftarrow \texttt{add}\ t\ s; \mathsf{return}\ ()$$

(because the postcondition of $\texttt{fresh\_str}\ t$ ensures the precondition of $\texttt{add}\ t\ s$), the erroneous program in Figure 1, where a string produced by `fresh_str` is added to a string table twice, is rejected:

$$\lambda t{:}\mathsf{tbl}.\mathsf{do}\ s \leftarrow \texttt{fresh\_str}\ t; \_ \leftarrow \texttt{add}\ t\ s; \_ \leftarrow \texttt{add}\ t\ s; \mathsf{return}\ ()$$

because the pre- and postcondition of $\texttt{add}\ t\ s$ do not coincide. We can execute the erroneous program by inserting a run-time check which confirms that s is not contained in t immediately before the second call to `add`, but the run-time check will fail.

2.3 Region-Based Effect System

As mentioned in Section 1.3, we have to restrict refinements to pure computations and pre- and postconditions to read-only computations. One naive approach to it is to restrict refinements to terms and pre- and postconditions to computations involving with only return and dereference constructs (binds may introduce assignments as do-expressions). However, it limits the expressive power of contracts excessively. For example, it would disallow contracts to use imperative libraries—e.g., hash tables and regular expression matching [32]—and interface functions for abstracted mutable data structures.

Our solution to relaxing the restriction while retaining the expressive power of contracts is to introduce an effect system with locally scoped *regions* [31, 60]. The effect system accepts pure computations, which manipulate only locally allocated references, in refinement types and read-only computations, which do not apply assignment to references allocated in programs, in pre- and postconditions of Hoare types.

To observe whether contracts are independent of references in a program, our effect system tracks which program point references are allocated at, using a region, which intuitively identifies a program point, in a standard manner [8, 31, 60]. Regions, denoted by r, are introduced by let-region $\nu r.\ c$ [8, 60], which computes c under the local region r. If c manipulates only references allocated in r, it appears to be pure for the context surrounding the let-region. Similarly, if it does not write data to references allocated in other regions, it appears to be read-only. Which regions references are allocated at is embedded into reference types, as usual in the work on region calculi with states [8, 31]. Reference types $\mathsf{Ref}_r\ T$ denote references which point to contents of T and are allocated at r.

To ensure that refinements are pure and pre- and postconditions are read-only, we track what operations contracts apply to references as effects and check that forbidden effects are not involved. As is standard in monadic languages [36, 64, 66], we embed effect information into types for computations, that is, Hoare types. In $\lambda^{\mathsf{H}}_{\mathsf{ref}}$, Hoare types actually take the form $\{A_1\}x{:}T\{A_2\}^{\langle\gamma_{\mathtt{r}},\gamma_{\mathtt{w}}\rangle}$, where $\gamma_{\mathtt{r}}$ and $\gamma_{\mathtt{w}}$ are sets of regions whose references are readable and writable in computations of the Hoare type, respectively. Refinements have to be typed at $\{\top\}x{:}\mathsf{bool}\{\top\}^{\langle\emptyset,\emptyset\rangle}$, where: $\gamma_{\mathtt{r}} = \gamma_{\mathtt{w}} = \emptyset$ means that the refinements have to be pure; $A_1 = \top$ that the refinements cannot suppose any precondition, since values of the refinement type must be copyable to any contexts; and $A_2 = \top$ that it is not needed to guarantee any postcondition. Pre- and postconditions have to be typed at $\{A_1\}x{:}\mathsf{bool}\{\top\}^{\langle\gamma_{\mathtt{r}},\emptyset\rangle}$ for some A_1 and $\gamma_{\mathtt{r}}$, where: $\gamma_{\mathtt{w}} = \emptyset$ means that the conditions have to be read-only—in other words, they are allowed to read data from references in regions $\gamma_{\mathtt{r}}$; and the conditions can assume some A_1 established before checking them.

Moreover, to enhance reusability of program components, we introduce region abstractions [31, 60], which are abstracted over region variables, so they can be used in any contracts. Region polymorphic types $\forall r.\,T$ are types for region abstractions.

Example The effect system allows many efficient algorithms and data structures with mutable references to be reused in contracts as well as programs. For example, using the let-region construct, an efficient implementation of regular expression matching, which often rests on mutable references, would be given the following type because it would rest on only locally allocated references:

$$\mathsf{string} \to \mathsf{string} \to \{\top\}x{:}\mathsf{bool}\{\top\}^{\langle\emptyset,\emptyset\rangle},$$

where the first and second arguments mean regular expressions and target strings, respectively. The Hoare type means that the regular expression matching is pure, so contracts can use it.

Moreover, the effect system enables contracts to mention "hidden states" of an abstracted data structure via interface functions for it. For example, it accepts the types given to functions `mem`, `fresh_str`, and `add` for abstracted string tables in Section 2.2, using some r:

$$\begin{array}{l}\texttt{mem} : \mathsf{tbl} \to \mathsf{string} \to \{\top\}x{:}\mathsf{bool}\{\top\}^{\langle\{r\},\emptyset\rangle}\\ \texttt{fresh\_str} : t{:}\mathsf{tbl} \to\\ \qquad\qquad \{\top\}s{:}\mathsf{string}\{y \leftarrow \texttt{mem}\ t\ s; \mathsf{return}\ \texttt{not}\ y\}^{\langle\{r\},\emptyset\rangle}\\ \texttt{add} : t{:}\mathsf{tbl} \to s{:}\mathsf{string} \to \{y \leftarrow \texttt{mem}\ t\ s; \mathsf{return}\ \texttt{not}\ y\}\\ \qquad\qquad\qquad\qquad\qquad\qquad x{:}\mathsf{unit}\\ \qquad\qquad\qquad\qquad\qquad\qquad \{y \leftarrow \texttt{mem}\ t\ s; \mathsf{return}\ y\}^{\langle\{r\},\{r\}\rangle}\end{array}$$

where read and write effects in both `mem` and `fresh_str` are $\{r\}$ and $\emptyset$, respectively, because they would dereference a string table pointer at r but would not update it, and both of the read and write effects in `add` are $\{r\}$ because it would add a string to a string table after dereference. Under the naive syntactic restriction, these types of `fresh_str` and `add` would be rejected because the contracts for them use bind constructs.

2.4 Run-time Checking of Pre- and Postconditions

To check pre- and postconditions of Hoare types, we provide a new computation construct, assertions. Assertion $\mathsf{assert}\ (c_1)^{\ell}; c_2$ first checks contract c_1 and then: if the contract checking succeeds, i.e., c_1 returns true, the remaining computation c_2 will be executed; otherwise, if it fails, an uncatchable exception $\Uparrow\ell$ will be raised. Perhaps assertions might appear to be able to check only preconditions, but they can also be used to check postconditions. For example, we can write a computation that first executes c_1 and then check postcondition c_2, using assertions, as $x \leftarrow \mathsf{do}\ c_1; \mathsf{assert}\ (c_2)^{\ell}; \mathsf{return}\ x$. For short, we write this computation as $c_1; \lambda x.\mathsf{assert}\ (c_2)^{\ell}$.

Example Let us consider whether a computation c satisfies the first contract in Section 2.2, that is, when reference x points to a prime number list, after computing it, the contents of x are still a prime number list and the computation returns the length of the list referred to by x. Using assertions, it is checked as follows:

$$\begin{array}{l}\mathsf{assert}\ (y \Leftarrow !x; \mathsf{return}\ (\texttt{for\_all}\ \texttt{prime}?\ y))^{\ell_1};\\ c;\\ \lambda z.\mathsf{assert}\ (y \Leftarrow !x;\\ \qquad\qquad \mathsf{return}\ (\texttt{for\_all}\ \texttt{prime}?\ y)\ \texttt{\&}\ (\texttt{length}\ y = z))^{\ell_2}\end{array}$$

Note that the contracts checked by these assertions match the pre- and postcondition of the Hoare type given in Section 2.2. When this computation is executed, first of all the precondition is checked; if the reference x does not refer to a prime number list, exception $\Uparrow\ell_1$ is raised; otherwise, the computation c is performed. If c terminates and returns a value, then, as a postcondition, whether reference x still points to a prime number list and whether the integer result of c is the length of the prime number list are checked. If the checking succeeds, the result of c is returned as the result of the whole computation; otherwise, $\Uparrow\ell_2$ is raised.

As a more interesting example, we consider an implementation of `add`, the Hoare type of which is given in Section 2.3. String tables are mutable references to string lists whose elements are pairwise distinct and we suppose that a concrete representation type of `tbl` is $\mathsf{Ref}_r\{l{:}\mathsf{string\ list} \mid \texttt{uniq}\ l\}$ where function `uniq`

Variables

x, y, z ::= term variables r, s ::= region variables
γ ::= finite sets of region variables ϱ ::= $\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \rangle$

Types

B ::= $\mathsf{bool} \mid \mathsf{unit} \mid \ldots$ A ::= $\top \mid A, c$
T ::= $B \mid x{:}T_1 \to T_2 \mid \{x{:}T \mid c\} \mid$
$\mathsf{Ref}_r\, T \mid \{A_1\}x{:}T\{A_2\}^{\varrho} \mid \forall r.T$

Constants, Terms, Commands, and Computations

k ::= $\mathsf{true} \mid \mathsf{false} \mid () \mid \ldots$
e ::= $x \mid k \mid op(e_1, \ldots, e_n) \mid \lambda x{:}T.e \mid \langle T_1 \Leftarrow T_2 \rangle^{\ell} \mid$
$e_1\, e_2 \mid e_1 == e_2 \mid \mathsf{do}\, c \mid \lambda r.e \mid e\{r\}$
d ::= $\mathsf{ref}_r\, e \mid !e \mid e_1 := e_2$
c ::= $\mathsf{return}\ e \mid x \leftarrow e_1; c_2 \mid x \Leftarrow d_1; c_2 \mid \nu r.\, c \mid \mathsf{assert}\,(c_1)^{\ell}; c_2$

Figure 2. Program syntax in $\lambda^{\mathsf{H}}_{\mathsf{ref}}$.

returns whether each element of a given list is unique in it. Then, the implementation of add can be given as follows:

$$
\begin{array}{l}
\lambda t{:}\mathsf{tbl}.\lambda s{:}\mathsf{string}.\ \mathsf{do} \\
\quad l \Leftarrow !t; \\
\quad \mathsf{let}\, l' = \langle \{l'{:}\mathsf{string\ list} \mid \mathtt{uniq}\, l'\} \Leftarrow \mathsf{string\ list} \rangle^{\ell_1}\, (s :: l); \\
\quad \_ \Leftarrow t := l'; \\
\quad \mathsf{return}\, (); \\
\quad \lambda \_.\mathsf{assert}\, (y \leftarrow \mathtt{mem}\, t\, s; \mathsf{return}\, y)^{\ell_2}
\end{array}
$$

Here, $\mathsf{let}\, x = e_1; c_2$ is an abbreviation of $x \leftarrow \mathsf{do}\, \mathsf{return}\, e_1; c_2$. This function accepts a string table t and a string s and returns a do-expression that adds s to t. The do-expression first dereferences t and obtains a list l whose elements are distinct from each other. Then, it checks with a cast that the new string s is fresh for l, and updates the string table with the new list if the check succeeds. Finally, the postcondition is checked by the assertion, in which $y \leftarrow \mathtt{mem}\, t\, s; \mathsf{return}\, y$ states that t contains s. This function involves two run-time checks: the cast $\langle \{l'{:}\mathsf{string\ list} \mid \mathtt{uniq}\, l'\} \Leftarrow \mathsf{string\ list} \rangle^{\ell_1}$ and the assertion $\mathsf{assert}\, (y \leftarrow \mathtt{mem}\, t\, s; \mathsf{return}\, y)^{\ell_2}$. It is obvious that the latter check always succeeds while whether the former succeeds rests on whether string s is not contained in table t before calling add: if s is *not* in t, the check succeeds; otherwise, it fails. Since s is passed by clients of add, we require them to pass strings which do not occur in t.[1] As a result, the Hoare type of add is given as in Section 2.3.

Contrary to the fact that add ensures the postcondition by assertions, its client does the precondition. For example, a function that extends a given string table with the string "foo" is written as:

$$
\begin{array}{l}
\lambda t{:}\mathsf{tbl}.\ \mathsf{do}\, \mathsf{assert}\, (y \leftarrow \mathtt{mem}\, t\, \texttt{"foo"}; \mathsf{return}\, \mathtt{not}\, y)^{\ell}; \\
\qquad\qquad \_ \leftarrow \mathtt{add}\, t\, \texttt{"foo"};\ \ \mathsf{return}\, ()
\end{array}
$$

where the precondition of add is checked before calling it.

Fortunately, we do not need the precondition check if it is ensured by the preceding computation. For example, the example in Section 2.2 shows that we can omit checking the precondition of add immediately after calling fresh_str since it is ensured by the postcondition of fresh_str.

3. Syntax

We show the program syntax of $\lambda^{\mathsf{H}}_{\mathsf{ref}}$ in Figure 2. The syntax uses various metavariables: B ranges over base types, A lists of pre- and postconditions of computations, T types, k constants, e terms, d commands, c computations. We use x, y, z, etc. as term variables, r and s as region variables, γ as sets of region variables, and ϱ as pairs of region sets. We write $\varrho_1 \cup \varrho_2$ for the element-wise union of ϱ_1 and ϱ_2, $\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \rangle \uplus \gamma$ for $\langle \gamma_{\mathtt{r}} \uplus \gamma, \gamma_{\mathtt{w}} \uplus \gamma \rangle$, where $\uplus$ is the union operation defined only when the given two region sets are disjoint, and γ, r for $\gamma \uplus \{r\}$.

Types offer base types, dependent function types, and refinement types from usual manifest calculi, in addition to reference types and Hoare types, which are already described in Section 2.2 in detail. We do not fix base types but assume bool and unit at least for contracts and assignment. Function types $x{:}T \to T'$, refinement types $\{x{:}T \mid c\}$, and Hoare types $\{A_1\}x{:}T\{A_2\}^{\varrho}$ bind variable x in T', c, and A_2, respectively. Region polymorphic types $\forall r.T$ bind region r in T.

Terms are usual lambda terms with constants, casts, pointer equality tests, do-expressions, region abstractions and applications. Constants include, at least, Boolean values true and false and the unit value $()$. A pointer equality test expression $e_1 == e_2$ returns whether two pointers e_1 and e_2 are equal. $\lambda r.e$, where r is bound in e, abstracts regions and $e\{r\}$ applies region abstraction e to region r.

Computations consist of return, bind, operations on references, let-region, and assertion. Region r in $\mathsf{ref}_r\, e$ specifies where a memory cell storing e is allocated. $x \leftarrow e_1; c$ and $x \Leftarrow d_1; c$ bind variable x in c; $\nu r.\, c$ binds r in c.

Finally, we introduce usual notations. We write $[\, e'/x \,]\, e$ for capture avoiding substitution of e' for x in e. α-equivalent terms are identified and a term without free term variables is said to be closed. These notions are applied to other syntactic categories such as computations and types. We use function $\mathit{frv}\,(c)$, which returns the set of free region variables in computation c. As shorthand, we write: $T_1 \to T_2$ for $x{:}T_1 \to T_2$ when x does not occur free in T_2; and $\mathsf{let}\, x = e_1\, \mathsf{in}\, e_2$ for $(\lambda x{:}T.e_2)\, e_1$ where T is an adequate type.

4. Type System

This section introduces a type system for programs in $\lambda^{\mathsf{H}}_{\mathsf{ref}}$. The goal of the type system is to guarantee that well-typed programs can't go wrong *except for contract violations*, i.e., they evaluate to values, raise exceptions by cast or assertion failure, or diverge. The type system is not strong enough to exclude possible contract violations; however, it *does* guarantee that result values and result stores of well-typed programs satisfy the contracts on their types.

The type system has five judgments defined mutually recursively by rules in Figure 3: typing context well-formedness $\gamma \vdash \Gamma$, type well-formedness $\gamma; \Gamma \vdash T$, assertion well-formedness $\gamma; \Gamma \vdash^{\varrho} A$ (where ϱ stands for effects which may occur in computations of A), term typing judgment $\gamma; \Gamma \vdash e : T$, and computation typing judgment $\gamma; \Gamma \vdash c : \{A_1\}x{:}T\{A_2\}^{\varrho}$. Typing contexts Γ, sequences of term and region variable declarations, are defined in a standard manner:

$$\Gamma ::= \emptyset \mid \Gamma, x{:}T \mid \Gamma, r$$

Region variables declared in Γ are introduced by region abstraction, whereas those in γ are by let-region. The two kinds of region variables are distinguished to reflect the fact that ν-bound variables are never replaced by substitution. We assume that term and region variables declared in typing contexts are distinct and write $\mathit{regions}\,(\Gamma)$ for the set of region variables declared in Γ.

Inference rules for typing context and type well-formedness judgments are standard [3, 21, 48, 49] or straightforward except (WF_REFINE) and (WF_HOARE). The rule (WF_REFINE) means that refinements must be pure ($\langle \emptyset, \emptyset \rangle$) and cannot assume but do not have to ensure anything. The rule (WF_HOARE) states that the pre-

[1] The cast is still left in well-typed programs because static verification of contracts are "post facto" [3].

$\boxed{\gamma \vdash \Gamma}$ $\boxed{\gamma; \Gamma \vdash T}$ $\boxed{\gamma; \Gamma \vdash^{\varrho} A}$ **Well-Formedness Rules for Typing Contexts, Types, and Assertions**

$$\frac{}{\gamma \vdash \emptyset}\ \text{WF\_EMPTY} \qquad \frac{\gamma \vdash \Gamma \quad \gamma; \Gamma \vdash T}{\gamma \vdash \Gamma, x{:}T}\ \text{WF\_EXTENDVAR} \qquad \frac{\gamma \vdash \Gamma \quad r \notin \gamma}{\gamma \vdash \Gamma, r}\ \text{WF\_EXTENDREGION} \qquad \frac{\gamma \vdash \Gamma}{\gamma; \Gamma \vdash B}\ \text{WF\_BASE}$$

$$\frac{\gamma; \Gamma \vdash T_1 \quad \gamma; \Gamma, x{:}T_1 \vdash T_2}{\gamma; \Gamma \vdash x{:}T_1 \to T_2}\ \text{WF\_FUN} \qquad \frac{\gamma; \Gamma \vdash T \quad \gamma; \Gamma, x{:}T \vdash c : \{\top\} y{:}\mathsf{bool}\{\top\}^{\langle \emptyset, \emptyset \rangle}}{\gamma; \Gamma \vdash \{x{:}T \mid c\}}\ \text{WF\_REFINE}$$

$$\frac{\gamma; \Gamma \vdash T \quad r \in \gamma \cup \mathit{regions}\,(\Gamma)}{\gamma; \Gamma \vdash \mathsf{Ref}_r\, T}\ \text{WF\_REF} \qquad \frac{\gamma; \Gamma \vdash^{\varrho} A_1 \quad \gamma; \Gamma \vdash T \quad \gamma; \Gamma, x{:}T \vdash^{\varrho} A_2}{\gamma; \Gamma \vdash \{A_1\} x{:}T \{A_2\}^{\varrho}}\ \text{WF\_HOARE} \qquad \frac{\gamma; \Gamma, r \vdash T}{\gamma; \Gamma \vdash \forall r.\, T}\ \text{WF\_RFUN}$$

$$\frac{\gamma_{\mathtt{r}} \cup \gamma_{\mathtt{w}} \subseteq \gamma \cup \mathit{regions}\,(\Gamma)}{\gamma; \Gamma \vdash^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \rangle} \top}\ \text{WF\_EMPTYASSERT} \qquad \frac{\gamma; \Gamma \vdash^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \rangle} A \quad \gamma; \Gamma \vdash c : \{A\} x{:}\mathsf{bool}\{\top\}^{\langle \gamma_{\mathtt{r}}, \emptyset \rangle}}{\gamma; \Gamma \vdash^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \rangle} A, c}\ \text{WF\_EXTENDASSERT}$$

$\boxed{\gamma; \Gamma \vdash e : T}$ **Term Typing Rules**

$$\frac{\gamma \vdash \Gamma \quad x{:}T \in \Gamma}{\gamma; \Gamma \vdash x : T}\ \text{T\_VAR} \qquad \frac{\gamma \vdash \Gamma}{\gamma; \Gamma \vdash k : \mathit{ty}\,(k)}\ \text{T\_CONST} \qquad \frac{\gamma \vdash \Gamma \quad \mathit{ty}\,(\mathit{op}) = x_1{:}T_1 \to \ldots \to x_n{:}T_n \to T \quad \forall\, i \in \{1, \ldots, n\}.\, \gamma; \Gamma \vdash e_i : [\, e_1/x_1, \ldots, e_{i-1}/x_{i-1}\,]\, T_i}{\gamma; \Gamma \vdash \mathit{op}(e_1, \ldots, e_n) : [\, e_1/x_1, \ldots, e_n/x_n\,]\, T}\ \text{T\_OP}$$

$$\frac{\gamma; \Gamma, x{:}T_1 \vdash e : T_2}{\gamma; \Gamma \vdash \lambda x{:}T_1.e : x{:}T_1 \to T_2}\ \text{T\_ABS} \qquad \frac{\gamma; \Gamma \vdash T_1 \quad \gamma; \Gamma \vdash T_2 \quad T_1 \parallel T_2}{\gamma; \Gamma \vdash \langle T_1 \Leftarrow T_2 \rangle^{\ell} : T_2 \to T_1}\ \text{T\_CAST} \qquad \frac{\gamma; \Gamma \vdash c : \{A_1\} x{:}T \{A_2\}^{\varrho}}{\gamma; \Gamma \vdash \mathsf{do}\, c : \{A_1\} x{:}T \{A_2\}^{\varrho}}\ \text{T\_DO}$$

$$\frac{\gamma; \Gamma \vdash e_1 : x{:}T_1 \to T_2 \quad \gamma; \Gamma \vdash e_2 : T_1}{\gamma; \Gamma \vdash e_1\, e_2 : [\, e_2/x\,]\, T_2}\ \text{T\_APP} \qquad \frac{\gamma; \Gamma \vdash e_1 : \mathsf{Ref}_r\, T_1 \quad \gamma; \Gamma \vdash e_2 : \mathsf{Ref}_s\, T_2 \quad T_1 \parallel T_2}{\gamma; \Gamma \vdash e_1 == e_2 : \mathsf{bool}}\ \text{T\_EQ}$$

$$\frac{\gamma; \Gamma, r \vdash e : T}{\gamma; \Gamma \vdash \lambda r.e : \forall r.\, T}\ \text{T\_RABS} \qquad \frac{\gamma; \Gamma \vdash e : \forall s.\, T \quad r \in \gamma \cup \mathit{regions}\,(\Gamma)}{\gamma; \Gamma \vdash e\{r\} : [\, r/s\,]\, T}\ \text{T\_RAPP}$$

$\boxed{\gamma; \Gamma \vdash c : \{A_1\} x{:}T \{A_2\}^{\varrho}}$ **Computation Typing Rules**

$$\frac{\gamma; \Gamma \vdash e : T \quad \gamma; \Gamma, x{:}T \vdash^{\varrho} A}{\gamma; \Gamma \vdash \mathsf{return}\, e : \{[\, e/x\,]\, A\} x{:}T \{A\}^{\varrho}}\ \text{CT\_RETURN} \qquad \frac{\gamma; \Gamma \vdash e_1 : \{A_1\} y{:}T_1 \{A_3\}^{\varrho_1} \quad \gamma; \Gamma \vdash \{A_1\} x{:}T_2 \{A_2\}^{\varrho_1 \cup \varrho_2} \quad \gamma; \Gamma, y{:}T_1 \vdash c_2 : \{A_3\} x{:}T_2 \{A_2\}^{\varrho_2}}{\gamma; \Gamma \vdash y \leftarrow e_1; c_2 : \{A_1\} x{:}T_2 \{A_2\}^{\varrho_1 \cup \varrho_2}}\ \text{CT\_BIND}$$

$$\frac{\gamma; \Gamma \vdash e : T' \quad \gamma; \Gamma \vdash \{A_1\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \cup \{r\} \rangle} \quad \gamma; \Gamma, y{:}\mathsf{Ref}_r\, T' \vdash c : \{A_1\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \rangle}}{\gamma; \Gamma \vdash y \Leftarrow \mathsf{ref}_r e; c : \{A_1\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \cup \{r\} \rangle}}\ \text{CT\_NEW} \qquad \frac{\gamma; \Gamma \vdash e : \mathsf{Ref}_r\, T' \quad \gamma; \Gamma \vdash \{A_1\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}} \cup \{r\}, \gamma_{\mathtt{w}} \rangle} \quad \gamma; \Gamma, y{:}T' \vdash c : \{A_1\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \rangle}}{\gamma; \Gamma \vdash y \Leftarrow !e; c : \{A_1\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}} \cup \{r\}, \gamma_{\mathtt{w}} \rangle}}\ \text{CT\_DEREF}$$

$$\frac{\gamma; \Gamma \vdash e_1 : \mathsf{Ref}_r\, T' \quad \gamma; \Gamma \vdash \{A_1\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \cup \{r\} \rangle} \quad \gamma; \Gamma \vdash e_2 : T' \quad \gamma; \Gamma, y{:}\mathsf{unit} \vdash c_3 : \{\top\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \rangle}}{\gamma; \Gamma \vdash y \Leftarrow e_1 := e_2; c_3 : \{A_1\} x{:}T \{A_2\}^{\langle \gamma_{\mathtt{r}}, \gamma_{\mathtt{w}} \cup \{r\} \rangle}}\ \text{CT\_ASSIGN} \qquad \frac{\gamma; \Gamma \vdash c_2 : \{A_1, c_1\} x{:}T \{A_2\}^{\varrho}}{\gamma; \Gamma \vdash \mathsf{assert}\,(c_1)^{\ell}; c_2 : \{A_1\} x{:}T \{A_2\}^{\varrho}}\ \text{CT\_ASSERT}$$

$$\frac{\gamma; \Gamma \vdash \{A_1\} x{:}T \{A_2\}^{\varrho} \quad \gamma, r; \Gamma \vdash c : \{A_1\} x{:}T \{A_2\}^{\varrho \uplus \{r\}}}{\gamma; \Gamma \vdash \nu r.\, c : \{A_1\} x{:}T \{A_2\}^{\varrho}}\ \text{CT\_LETREGION} \qquad \frac{\gamma; \Gamma \vdash c : \{A_1'\} x{:}T \{A_2'\}^{\varrho} \quad A_1' \subseteq A_1 \quad A_2 \subseteq A_2' \quad \gamma; \Gamma \vdash \{A_1\} x{:}T \{A_2\}^{\varrho}}{\gamma; \Gamma \vdash c : \{A_1\} x{:}T \{A_2\}^{\varrho}}\ \text{CT\_WEAK}$$

Figure 3. Type system for $\lambda^{\mathsf{H}}_{\mathsf{ref}}$.

and postcondition of a Hoare type with effect ϱ should be no more effectful than ϱ.

Assertion well-formedness is derived by two rules. In the rule (WF_EXTENDASSERT), which is applied to append another condition c to a sequence A, the second premise means that: the appended condition c can assume that the preceding conditions A hold, has to be read-only, and does not have to ensure anything.

Typing rules for terms are syntax-directed and almost standard. The rules (T_CONST) and (T_OP) use metafunction ty, which returns the type of each constant and each operator. The rule (T_APP) substitutes an argument term e_2 for variable x bound in return type T_2. This substitution causes no problems because terms in our calculus are almost pure—i.e., they return values or raise exceptions—and it is known that some computational effects, such as raise of uncatchable exceptions and nontermination, do not cause problems in manifest contracts [3, 49]. The rule (T_CAST) requires the source and target types of a well-typed cast to be well formed and *compatible* [22, 30]. Intuitively, a type T_1 is compatible with another type T_2 when they are identified after dropping all contracts and region information. Formally, compatibility $\parallel$ is the congruence satisfying $\{x{:}T \mid c\} \parallel T$, $\mathsf{Ref}_r\, T \parallel \mathsf{Ref}_s\, T$, and $\{A_{11}\} x{:}T \{A_{12}\}^{\varrho_1} \parallel \{A_{21}\} x{:}T \{A_{22}\}^{\varrho_2}$. The rule (T_EQ) allows terms typed at compatible reference types to be compared because the same pointer can be cast to different (reference) types; note that $T_1 \parallel T_2$ iff $\mathsf{Ref}_r\, T_1 \parallel \mathsf{Ref}_s\, T_2$ for any r and s. The compatibility check in (T_CAST) and (T_EQ) reports casts that always fail and equality tests that always return false without evaluating.

Computation typing rules are more interesting. The typing rule (CT_RETURN) gives return e the Hoare type $\{[\, e/x\,]\, A\} x{:}T \{A\}^{\varrho}$ where T is the type of e and ϱ is effects which may occur in A. If

$[\,e/x\,]\,A$ is satisfied, return e results in a store satisfying A because x in A denotes the value of e and return e does not manipulate stores. The rule (CT_BIND) for $x \leftarrow e_1; c_2$ requires e_1 to evaluate to do c_1 (if terminates) and allows the remaining computation c_2 to refer to, by x, the result of computing the suspended computation c_1. Since $x \leftarrow e_1; c_2$ involves effects of c_1 and c_2, the effect of the result type is the union $\varrho_1 \cup \varrho_2$ of effects of e_1 and c_2. Satisfaction of the precondition of c_2 has to be promised by the postcondition in the type of e_1 because c_2 will be computed immediately after executing c_1. Moreover, (CT_BIND) demands that the result type $\{A_1\}x{:}T_2\{A_2\}^{\varrho_1 \cup \varrho_2}$ be well formed under the typing context Γ without y since y is a variable locally bound by bind—other typing rules need similar conditions. The typing rules (CT_NEW), (CT_DEREF), and (CT_ASSIGN) are applied to a computation with memory allocation, dereference, and assignment, respectively. In these rules, the corresponding effect is added to the result Hoare type. The first two rules are not surprising. The rule (CT_ASSIGN) says that the remaining computation c_3 cannot assume anything (hence the empty condition) because the assignment could invalidate the precondition A_1—for example, the condition $x \Leftarrow !e; \mathsf{return}\ x\ =\ 0$ does not hold after executing $e \coloneqq 2$. In general, we do not know which conditions still hold and which conditions do not, so we suppose the worst-case scenario, that is, that all conditions are invalidated. In fact, we can do better because if references manipulated by assignment are allocated at r, conditions which do not involve effects including r will not be invalidated—we discuss this recovery of preconditions of the remaining computation in Section 7. Finally, the result of an assignment is the unit value, so the typing context in the third premise of (CT_ASSIGN) includes y:unit. The rule (CT_LETREGION) is applied to let-region $\nu r.\, c$ and requires the body c to be well typed under the region set including region r. The well-formedness of the result Hoare type under the region set without r ensures that access to r is not observed by the context. An assertion $\mathsf{assert}\,(c_1)^\ell; c_2$ is typed by (CT_ASSERT), which allows the remaining computation c_2 to assume that the condition c_1 holds since its satisfaction is ensured at run time; well typedness of c_1 is ensured by the preceding typing derivation. The last rule (CT_WEAK) allows strengthening preconditions, weakening postconditions, and permuting them. The partial order $A_1 \subseteq A_2$ over conditions means that, for any c in A_1, there exist some A and A' such that $A_2 = A, c, A'$. Although (CT_WEAK) manipulates conditions syntactically, static verification in Section 7 enables more flexible manipulation. For example, using function `size` which returns the size of a table, the technique in that section allows computations of $\{\top\}t{:}\mathsf{tbl}\{y \leftarrow \mathtt{mem}\ t\ \mathtt{"foo"}; \mathsf{return}\ y\}^\varrho$ to be regarded as ones of $\{\top\}t{:}\mathsf{tbl}\{y \leftarrow \mathtt{size}\ t; \mathsf{return}\ y > 0\}^\varrho$ (if it can be proven that the former postcondition implies the latter), which cannot be derived from (CT_WEAK). Finally, we make a remark about effect weakening—well typed computations can be given a Hoare type with more effects. Although there is no rule for effect weakening, it is admissible:

Lemma 1 (Effect Weakening)**.** *If* $\langle \gamma_\mathtt{r}, \gamma_\mathtt{w} \rangle \subseteq \langle \gamma_\mathtt{r}', \gamma_\mathtt{w}' \rangle$ *and* $\gamma_\mathtt{r}', \gamma_\mathtt{w}' \subseteq \gamma \cup$ *regions* (Γ) *and* $\gamma; \Gamma \vdash c : \{A_1\}x{:}T\{A_2\}^{\langle \gamma_\mathtt{r}, \gamma_\mathtt{w} \rangle}$*, then* $\gamma; \Gamma \vdash c : \{A_1\}x{:}T\{A_2\}^{\langle \gamma_\mathtt{r}', \gamma_\mathtt{w}' \rangle}$.

5. Semantics

In this section, we define small-step call-by-value operational semantics for $\lambda^{\mathsf{H}}_{\mathsf{ref}}$. The semantics mainly consists of two relations, reduction relation $\rightsquigarrow$ for terms and computation relation $\longrightarrow$ for computations. In what follows, we begin with describing intuition about run-time checking introduced in this work—casts for reference types and Hoare types, and contract checks with local regions and local stores. Then, we formalize the semantics after extending the program syntax with run-time terms and computations.

5.1 Run-time Checking for References and do-Expressions

In this section, we outline how casts for reference types and Hoare types work. Casts for other types are similar to the previous work [3, 18, 30, 48, 49].

Casts between reference types with the same region generate *reference guards* $T_1 \Leftarrow^\ell T_2 : v$, which are a key construct in the earlier work on dynamic checking with references [13, 19, 24, 55]:

$$\langle \mathsf{Ref}_r\, T_1 \Leftarrow \mathsf{Ref}_r\, T_2 \rangle^\ell\, v \rightsquigarrow T_1 \Leftarrow^\ell T_2 : v \qquad \text{R\_REF}$$

Otherwise, if the regions are different, the cast fails:

$$\langle \mathsf{Ref}_r\, T_1 \Leftarrow \mathsf{Ref}_s\, T_2 \rangle^\ell\, v \rightsquigarrow \Uparrow \ell \quad (\text{where } r \neq s) \quad \text{R\_REFFAIL}$$

Reference guards are "proxies" to monitor dereference and assignment operations at run time so that they behave as the target reference type. When reference guard $T_1 \Leftarrow^\ell T_2 : v$ is dereferenced, the run-time system checks that the contents of v can work as T_1:

$$x \Leftarrow !(T_1 \Leftarrow^\ell T_2 : v); c \longrightarrow y \Leftarrow !v; \mathsf{let}\, x = \langle T_1 \Leftarrow T_2 \rangle^\ell\, y; c.$$

When it is assigned a value v' of T_1, it is checked that v' can be assigned to v:

$$x \Leftarrow (T_1 \Leftarrow^\ell T_2 : v) \coloneqq v'; c \longrightarrow x \Leftarrow v \coloneqq \langle T_2 \Leftarrow T_1 \rangle^\ell\, v'; c.$$

Casts between Hoare types are similar to casts between function types [16, 18] in the sense that computations are functions over states. Since a cast is a term-level construct, the result of cast application $\langle \{A_{11}\}x{:}T_1\{A_{12}\}^{\varrho_1} \Leftarrow \{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho_2} \rangle^\ell\, v$ (where v's type is $\{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho_2}$) is a do-expression, which triggers execution of v with additional checks: it is ensured that the do-expression does not hide the effects of v; the precondition A_{21} of v is checked before its execution; the result of v is cast back to T_1 from T_2; and, finally, the postcondition A_{12} is checked. Formally, the cast application reduces as follows:

$$\begin{array}{l}\langle \{A_{11}\}x{:}T_1\{A_{12}\}^{\varrho_1} \Leftarrow \{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho_2} \rangle^\ell\, v \rightsquigarrow \\ \quad \mathsf{do}\ \mathsf{assert}\,(A_{21})^\ell; y \leftarrow v; \mathsf{let}\, x = \langle T_1 \Leftarrow T_2 \rangle^\ell\, y; \\ \qquad \mathsf{assert}\,(A_{12})^\ell; \mathsf{return}\, x \quad (\text{where } \varrho_2 \subseteq \varrho_1) \quad \text{R\_HOARE}\end{array}$$

where y is a fresh variable and notation $\mathsf{assert}\,(A)^\ell; c$ means, for $A = c_1, ..., c_n$, $\mathsf{assert}\,(c_1)^\ell; ...; \mathsf{assert}\,(c_n)^\ell; c$. If ϱ_1 is not more effectful than ϱ_2, the cast fails:

$$\begin{array}{l}\langle \{A_{11}\}x{:}T_1\{A_{12}\}^{\varrho_1} \Leftarrow \{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho_2} \rangle^\ell\, v \rightsquigarrow \Uparrow \ell \\ \qquad\qquad (\text{where } \varrho_2 \not\subseteq \varrho_1) \quad \text{R\_HOAREFAIL}\end{array}$$

5.2 Contract Checking with Local Stores

Effects for references at locally introduced regions in contracts must not be observed by programs, which is important especially for showing correctness of static verification in Section 7. Unfortunately, this request makes it difficult to apply the small-step operational semantics of the previous work on a region calculus [8], where reduction of let-regions changes program stores, to our work because it is unclear how to distinguish changes to program stores by programs and those by contracts.

Our approach to the request is to introduce *local* stores to intermediate states of contract checking and design a semantics where memory allocation and assignment operations with respect to locally introduced regions in contracts are applied to the local stores. Thanks to local stores, during contract checking, program stores do not change and, as a result, effects involved by contracts are not observed.

Formally, we introduce an expression of the form $\nu\gamma.\langle \mu \mid c \rangle$ (called checking state), where γ, μ, and c are a set of local regions, a local store, and a computation, respectively, to express

Stores, Values, Terms, Computations, and Checking States

$$
\begin{array}{rcll}
a, b & ::= & \text{memory addresses} & \mu \ ::= \ \{a_i@r_i \mapsto v_i\}_i \\
v & ::= & k \mid \lambda x{:}T.e \mid \langle T_1 \Leftarrow T_2 \rangle^{\ell} \mid \mathsf{do}\, c \mid \lambda r.e \mid & \\
 & & a@r \mid T_1 \Leftarrow^{\ell} T_2 : v & \\
e & ::= & \ldots \mid a@r \mid T_1 \Leftarrow^{\ell} T_2 : v \mid & \\
 & & \Uparrow\ell \mid \langle\!\langle \{x{:}T \mid c\}, e \rangle\!\rangle^{\ell} \mid \langle \{x{:}T \mid c\}, p, v \rangle^{\ell} & \\
c & ::= & \ldots \mid \Uparrow\ell \mid \langle \mathsf{assert}\,(c_1), p_2 \rangle^{\ell}; c_3 & p \ ::= \ \nu\gamma.\langle \mu \mid c \rangle
\end{array}
$$

Figure 4. Run-time syntax in $\lambda^{\mathsf{H}}_{\mathsf{ref}}$.

a program state during contract checking. Starting with checking a contract c, the run-time system generates an initial checking state $\nu\emptyset.\langle\emptyset \mid c\rangle$, where there are no locally introduced regions and no locally allocated references, and then starts computing c. During the check, newly introduced regions are added to the local regions, new memory cells are allocated to the local store, and dereference and assignment for memory cells at the local regions are applied to the local store. If c results in $\mathsf{return}\,\mathsf{true}$, the check succeeds; if c results in $\mathsf{return}\,\mathsf{false}$, it fails.

5.3 Definition

5.3.1 Run-time Syntax

We show the run-time syntax in Figure 4. We use a and b to denote memory addresses. The definition of values v is not surprising except that a memory address is paired with a region to indicate where region a cell is allocated. Stores, ranged over by μ, are finite mappings from pairs of a memory address and a region to closed values. We write $\mu_1 \uplus \mu_2$ for the concatenation of μ_1 and μ_2 with disjoint domains. Checking states $\nu\gamma.\langle\mu \mid c\rangle$, denoted by p, bind γ in μ and c.

The forms of run-time terms are straightforward except for the last two constructs, which represent intermediate states of refinement checking. A waiting check $\langle\!\langle \{x{:}T \mid c\}, e \rangle\!\rangle^{\ell}$, introduced by Sekiyama et al. [48] to prove a critical property of a manifest contract calculus with Belo et al.'s approach [3, 21], waits for evaluation of e before starting a check that the value of e satisfies the contract c. An active check $\langle \{x{:}T \mid c\}, p, v \rangle^{\ell}$ is verifying that the value v satisfies the refinement c; p is an intermediate state of a check which has been started by running $[\,v/x\,]\,c$. If the intermediate computation of p results in $\mathsf{return}\,\mathsf{true}$, the active check evaluates to v; otherwise, if it results in $\mathsf{return}\,\mathsf{false}$, $\Uparrow\ell$ will be raised.

Run-time computations have two additional constructs: exceptions $\Uparrow\ell$ and intermediate states $\langle \mathsf{assert}\,(c_1), p_2 \rangle^{\ell}; c_3$ during the check of assertion c_1: if the check succeeds, the remaining computation c_3 will be executed, and if it fails, $\Uparrow\ell$ will be raised.

5.3.2 Reduction

Reduction $\rightsquigarrow$, defined over closed run-time terms, is given by using rules shown at the top of Figure 5. The first three rules are standard in lambda calculi with call-by-value semantics, or straightforward. $[\![-]\!]$ in (R_OP) assigns a function over base type values to an operation op. Reduction of pointer equality tests uses $ungrd$ to peel off all reference guards:

$$ungrd\,(a@r) = a@r \quad ungrd\,(T_1 \Leftarrow^{\ell} T_2 : v) = ungrd\,(v)$$

The next several rules are cast reduction rules, some of which are similar to the ones in the previous work [48, 49]. The rule (R_RFUN) generates a region abstraction that wraps the target value, like cast reduction for type abstractions [3, 49]. The rule (R_FUN) produces a "function proxy" [6, 22], which applies the contravariant cast on argument types to an argument, passes its result to the original function, and applies the covariant cast on return types to the value returned by the original function. To avoid capture of variable x bound in the source return type, it is renamed with a fresh variable y. By (R_FORGET) and (R_PRECHECK), a cast application for refinement types first forgets all refinements in the source type and then reduces to a waiting check which verifies that the target value satisfies the outermost contract after checks of inner contracts. The side condition in (R_PRECHECK) makes the semantics deterministic. After checks of inner contracts, (R_CHECK) produces an active check to verify the outermost contract. The rule (R_CHECKING) reduces the active check by evaluating the contract checking state under the empty store—we see how checking states reduce later—and (R_BLAME) lifts up exceptions that happen during the check. If an active check succeeds, it returns the target value ((R_OK)); otherwise, it raises $\Uparrow\ell$ (R_FAIL).

5.3.3 Computation

Computation $\longrightarrow$, defined over pairs $\mu \mid c$ of a store and a closed run-time computation, is given by the rules in Figure 5 with several auxiliary rules to execute commands.

The rules (C_RED) and (C_COMPUT) are applied to reduce subterms and subcomputations of a computation, using computation contexts on terms, ranged over by C^{e}:

$$
\begin{array}{rcl}
E & ::= & [\,] \mid op(v_1, \ldots, v_n, E, e_1, \ldots, e_n) \mid E\ e_2 \mid v_1\ E \mid \\
 & & E == e_2 \mid v_1 == E \mid E\{r\} \mid \langle\!\langle \{x{:}T \mid c_1\}, E \rangle\!\rangle^{\ell} \\
D & ::= & \mathsf{ref}_r E \mid\ !E \mid E := e_2 \mid v_1 := E \\
C^{\mathsf{e}} & ::= & \mathsf{return}\ E \mid x \leftarrow E; c_2 \mid x \Leftarrow D; c_2
\end{array}
$$

The definition above means that subterms reduce from left to right.

Exceptions raised by subterms and subcomputations are lifted up by (C_RBLAME) and (C_CBLAME), respectively. The rule (C_CBLAME) also lifts up exceptions raised by contracts, using computation contexts on exceptions.

$$C^{\mathsf{1}} \ ::= \ x \leftarrow \mathsf{do}\,[\,]\,; c_2 \mid \langle \mathsf{assert}\,(c_1),\ \nu\gamma.\langle\mu \mid [\,]\rangle\rangle^{\ell}; c_3$$

The rule (C_RETURN) performs, when term e_1 of bind $x \leftarrow e_1; c_2$ reduces to a do-expression (by (C_RED)) and it returns a value v (by (C_COMPUT)), the remaining computation $[\,v/x\,]\,c_2$. Let-regions are lifted up to checking states by (C_REGION) in order to propagate newly created regions. For example, checking state $\nu\gamma.\langle\mu \mid \ldots \nu r.\, c' \ldots\rangle$, where the let-region is ready to be evaluated, goes on to $\nu\gamma.\langle\mu \mid \nu r.\, \ldots c' \ldots\rangle$ by (C_REGION). The rule (C_COMMAND), applied to execute commands, rests on the command relation $\rightarrowtail$, which transforms a command to a computation with an adequate action by rules shown in the middle of Figure 5. The first three command rules are standard except for the use of regions. The last two rules are used for dereference from and assignment to reference guards, as described in Section 5.1; here, variable x can be arbitrarily chosen since commands are closed.

Other computation rules are applied to check contracts with assertion. The rule (C_ASSERT) produces an intermediate checking state, which proceeds with the program store by (C_CHECKING) and: if the checking succeeds, the remaining computation c_2 starts (C_OK); otherwise, if it fails, $\Uparrow\ell$ is raised (C_FAIL).

Finally, computation of checking state $\nu\gamma.\langle\mu_1 \mid c_1\rangle$ under global store μ proceeds by two rules. (P_COMPUT) computes c_1 under the concatenation of the program store and the local store. The result store $\mu \uplus \mu_2$ means that the global store μ remains the same and, although dereference from μ is possible, memory allocation and assignment cannot take place on μ. The rule (P_REGION) adds regions bubbled up by (C_REGION) to the local regions. To see how these computation rules interact, let us consider contract checking. When a check of contract c happens by (R_CHECK) or (C_ASSERT), the check starts with $\nu\emptyset.\langle\emptyset \mid c\rangle$. During the check, when a let-region evaluates, it is lifted up by (C_REGION) and

$\boxed{e_1 \rightsquigarrow e_2}$ **Reduction Rules**

$op(k_1, \ldots, k_n) \rightsquigarrow [\![op]\!](k_1, \ldots, k_n)$ R_OP $\quad (\lambda x{:}T.e)\, v \rightsquigarrow [\,v/x\,]\, e$ R_BETA $\quad (\lambda r.e)\{s\} \rightsquigarrow [\,s/r\,]\, e$ R_RBETA
$v_1 == v_2 \rightsquigarrow \mathsf{true}$ (where $ungrd\,(v_1) = ungrd\,(v_2)$) R_EQ $\quad v_1 == v_2 \rightsquigarrow \mathsf{false}$ (where $ungrd\,(v_1) \neq ungrd\,(v_2)$) R_NEQ

$\langle B \Leftarrow B\rangle^\ell\, v \rightsquigarrow v$ R_BASE $\quad \langle \forall r.\, T_1 \Leftarrow \forall r.\, T_2\rangle^\ell\, v \rightsquigarrow \lambda r.\langle T_1 \Leftarrow T_2\rangle^\ell\,(v\{r\})$ R_RFUN
$\langle x{:}T_{11} \to T_{12} \Leftarrow x{:}T_{21} \to T_{22}\rangle^\ell\, v \rightsquigarrow \lambda x{:}T_{11}.\mathsf{let}\, y = \langle T_{21} \Leftarrow T_{11}\rangle^\ell\, x\, \mathsf{in}\, (\langle T_{12} \Leftarrow [\,y/x\,]\, T_{22}\rangle^\ell\,(v\, y))$ (where y is fresh) R_FUN
$\langle T_1 \Leftarrow \{x{:}T_2 \mid c_2\}\rangle^\ell\, v \rightsquigarrow \langle T_1 \Leftarrow T_2\rangle^\ell\, v$ R_FORGET $\quad \langle\{x{:}T_1 \mid c_1\} \Leftarrow T_2\rangle^\ell\, v \rightsquigarrow \langle\!\langle \{x{:}T_1 \mid c_1\}, \langle T_1 \Leftarrow T_2\rangle^\ell\, v \rangle\!\rangle^\ell$ (where $T_2 \neq \{y{:}T \mid c\}$ for any y, T, and c) R_PRECHECK
$\langle\!\langle \{x{:}T \mid c\}, v \rangle\!\rangle^\ell \rightsquigarrow \langle \{x{:}T \mid c\}, \nu\emptyset.\langle\emptyset \mid [\,v/x\,]\, c\rangle, v\rangle^\ell$ R_CHECK $\quad \langle \{x{:}T \mid c\}, \nu\gamma.\langle \mu \mid \Uparrow\ell'\rangle, v\rangle^\ell \rightsquigarrow \Uparrow\ell'$ R_BLAME
$\langle \{x{:}T \mid c\}, \nu\gamma.\langle \mu \mid \mathsf{return\ true}\rangle, v\rangle^\ell \rightsquigarrow v$ R_OK $\quad \langle \{x{:}T \mid c\}, \nu\gamma.\langle \mu \mid \mathsf{return\ false}\rangle, v\rangle^\ell \rightsquigarrow \Uparrow\ell$ R_FAIL
$\langle \{x{:}T \mid c\}, p, v\rangle^\ell \rightsquigarrow \langle \{x{:}T \mid c\}, p', v\rangle^\ell$ (where $\emptyset \mid p \hookrightarrow p'$) R_CHECKING

$\langle \mathsf{Ref}_r\, T_1 \Leftarrow \mathsf{Ref}_r\, T_2\rangle^\ell\, v \rightsquigarrow T_1 \Leftarrow^\ell T_2 : v$ R_REF $\quad \langle \mathsf{Ref}_r\, T_1 \Leftarrow \mathsf{Ref}_s\, T_2\rangle^\ell\, v \rightsquigarrow \Uparrow\ell$ (where $r \neq s$) R_REFFAIL
$\langle \{A_{11}\}x{:}T_1\{A_{12}\}^{\varrho_1} \Leftarrow \{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho_2}\rangle^\ell\, v \rightsquigarrow$
$\quad \mathsf{do\ assert}\,(A_{21})^\ell; y \leftarrow v; \mathsf{let}\, x = \langle T_1 \Leftarrow T_2\rangle^\ell\, y; \mathsf{assert}\,(A_{12})^\ell; \mathsf{return}\, x$ (where $\varrho_2 \subseteq \varrho_1$ and y is fresh) R_HOARE
$\langle \{A_{11}\}x{:}T_1\{A_{12}\}^{\varrho_1} \Leftarrow \{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho_2}\rangle^\ell\, v \rightsquigarrow \Uparrow\ell$ (where $\varrho_2 \not\subseteq \varrho_1$) R_HOAREFAIL

$\boxed{\mu_1 \mid d_1 \rightarrowtail \mu_2 \mid c_2}$ **Command Rules**

$\mu \mid \mathsf{ref}_r\, v \rightarrowtail \mu \uplus \{a@r \mapsto v\} \mid \mathsf{return}\, a@r$ C_NEW $\quad \mu \uplus \{a@r \mapsto v'\} \mid a@r := v \rightarrowtail \mu \uplus \{a@r \mapsto v\} \mid \mathsf{return}\,()$ C_ASSIGN
$\mu \mid !a@r \rightarrowtail \mu \mid \mathsf{return}\, \mu(a@r)$ C_DEREF $\quad \mu \mid !(T_1 \Leftarrow^\ell T_2 : v) \rightarrowtail \mu \mid x \Leftarrow !v; \mathsf{return}\,(\langle T_1 \Leftarrow T_2\rangle^\ell\, x)$ C_GDEREF
$\mu \mid (T_1 \Leftarrow^\ell T_2 : v_2) := v_1 \rightarrowtail \mu \mid x \Leftarrow v_2 := (\langle T_2 \Leftarrow T_1\rangle^\ell\, v_1); \mathsf{return}\,()$ C_GASSIGN

$\boxed{\mu_1 \mid c_1 \longrightarrow \mu_2 \mid c_2}$ **Computation Rules**

$$\frac{e_1 \rightsquigarrow e_2}{\mu \mid C^{\mathsf{e}}[\,e_1\,] \longrightarrow \mu \mid C^{\mathsf{e}}[\,e_2\,]}\ \text{C\_RED} \qquad \frac{\mu_1 \mid c_1 \longrightarrow \mu_2 \mid c_1'}{\mu_1 \mid x \leftarrow \mathsf{do}\, c_1; c_2 \longrightarrow \mu_2 \mid x \leftarrow \mathsf{do}\, c_1'; c_2}\ \text{C\_COMPUT}$$

$\mu \mid C^{\mathsf{e}}[\Uparrow\ell] \longrightarrow \mu \mid \Uparrow\ell$ C_RBLAME $\quad \mu \mid x \leftarrow \mathsf{do\ return}\, v_1; c_2 \longrightarrow \mu \mid [\,v_1/x\,]\, c_2$ C_RETURN
$\mu \mid C^{1}[\Uparrow\ell] \longrightarrow \mu \mid \Uparrow\ell$ C_CBLAME $\quad \mu \mid x \leftarrow (\mathsf{do}\, \nu r.\, c_1); c_2 \longrightarrow \mu \mid \nu r.\, (x \leftarrow \mathsf{do}\, c_1; c_2)$ (where $r \notin frv\,(c_2)$) C_REGION

$$\frac{\mu_1 \mid d_1 \rightarrowtail \mu_2 \mid c_1}{\mu_1 \mid x \Leftarrow d_1; c_2 \longrightarrow \mu_2 \mid x \leftarrow \mathsf{do}\, c_1; c_2}\ \text{C\_COMMAND} \qquad \frac{\mu \mid p_1 \hookrightarrow p_2}{\mu \mid \langle \mathsf{assert}\,(c_1), p_1\rangle^\ell; c_2 \longrightarrow \mu \mid \langle \mathsf{assert}\,(c_1), p_2\rangle^\ell; c_2}\ \text{C\_CHECKING}$$

$\mu \mid \mathsf{assert}\,(c_1)^\ell; c_2 \longrightarrow \mu \mid \langle \mathsf{assert}\,(c_1), \nu\emptyset.\langle\emptyset \mid c_1\rangle\rangle^\ell; c_2$ C_ASSERT $\quad \mu \mid \langle \mathsf{assert}\,(c_1), \nu\gamma.\langle \mu \mid \mathsf{return\ true}\rangle\rangle^\ell; c_2 \longrightarrow \mu \mid c_2$ C_OK
$\mu \mid \langle \mathsf{assert}\,(c_1), \nu\gamma.\langle \mu \mid \mathsf{return\ false}\rangle\rangle^\ell; c_2 \longrightarrow \mu \mid \Uparrow\ell$ C_FAIL

$\boxed{\mu \mid p_1 \hookrightarrow p_2}$ **Checking State Computation Rules**

$$\frac{\mu \uplus \mu_1 \mid c_1 \longrightarrow \mu \uplus \mu_2 \mid c_2}{\mu \mid \nu\gamma.\langle \mu_1 \mid c_1\rangle \hookrightarrow \nu\gamma.\langle \mu_2 \mid c_2\rangle}\ \text{P\_COMPUT} \qquad \mu \mid \nu\gamma.\langle \mu' \mid \nu r.\, c\rangle \hookrightarrow \nu(\gamma, r).\langle \mu' \mid c\rangle \quad \text{P\_REGION}$$

Figure 5. Operational semantics.

the region variable bound by it is added to the local region set in the checking state by (P_REGION). If the intermediate computation tries to create a reference or assign values to a reference, an adequate action is applied to the local store, not the global store, by (P_COMPUT). The intermediate computation can read data from references in either the global store or the local store because it evaluates in their concatenated store (P_COMPUT).

Top-level programs are executed in the form of checking states. We call computations c such that $\{r\}; \emptyset \vdash c : \{\top\}x{:}T\{A_2\}^{\langle\{r\},\{r\}\rangle}$ *programs*. A program c with designated region r, which stands for the global store, is executed by starting computation from $\nu\emptyset.\langle\emptyset \mid \nu r.\, c\rangle$ and program execution is performed by evaluation of checking states $\mu \mid p_1 \hookrightarrow^* p_2$, which means that there are $p_1', \ldots, p_n'$ such that $\mu \mid p_1 \hookrightarrow p_1', \mu \mid p_1' \hookrightarrow p_2', \ldots, \mu \mid p_n' \hookrightarrow p_2$. Thus, the program evaluation that the program c results in v is denoted by $\emptyset \mid \nu\emptyset.\langle\emptyset \mid \nu r.\, c\rangle \hookrightarrow^* \nu\gamma.\langle \mu \mid \mathsf{return}\, v\rangle$, where μ is the result global store.

6. Type Soundness

Following Belo et al. [3], which avoided semantic type soundness proofs, we show type soundness of $\lambda^{\mathsf{H}}_{\mathsf{ref}}$ via standard syntactic approaches (namely, progress and preservation [67]). In particular, we show that (1) a well-typed program returns a value, raises an exception, or diverges and (2) the result value and the result store of a well-typed program satisfy contracts on their types.

6.1 Run-Time Type System

To prove type soundness, we extend the type system in Section 4 to deal with run-time terms and computations. As usual, we introduce store typing contexts, ranged over by Σ, to record the type of the value that each reference points to. They are defined as follows:

$$\Sigma ::= \emptyset \mid \Sigma, a@r{:}T$$

We assume that references declared in store typing contexts are distinct and write Σ, Σ' for the concatenation of Σ and Σ'.

$\boxed{\Sigma;\gamma \vdash \Gamma}$ $\boxed{\Sigma;\gamma;\Gamma \vdash T}$ $\boxed{\Sigma;\gamma;\Gamma \vdash^{\varrho} A}$ $\boxed{\Sigma;\gamma;\Gamma \vdash e : T}$ **Run-time Term Typing Rules (Selected)**

$$\frac{\Sigma;\gamma \vdash \Gamma \quad a@r{:}T \in \Sigma \quad \Sigma;\gamma;\emptyset \vdash \mathsf{Ref}_r\, T}{\Sigma;\gamma;\Gamma \vdash a@r : \mathsf{Ref}_r\, T}\ \text{T\_ADDRESS} \qquad \frac{\Sigma;\gamma \vdash \Gamma \quad \Sigma;\gamma;\emptyset \vdash v : \mathsf{Ref}_r\, T_2 \quad T_1 \parallel T_2 \quad \Sigma;\gamma;\emptyset \vdash \mathsf{Ref}_r\, T_1}{\Sigma;\gamma;\Gamma \vdash T_1 \Leftarrow^{\ell} T_2 : v : \mathsf{Ref}_r\, T_1}\ \text{T\_GUARD}$$

$$\frac{\Sigma;\gamma \vdash \Gamma \quad \Sigma;\gamma;\emptyset \vdash T}{\Sigma;\gamma;\Gamma \vdash \Uparrow\ell : T}\ \text{T\_BLAME} \qquad \frac{\Sigma;\gamma \vdash \Gamma \quad \Sigma;\gamma;\emptyset \vdash \{x{:}T \mid c\} \quad \Sigma;\gamma;\emptyset \vdash e : T}{\Sigma;\gamma;\Gamma \vdash \langle\!\langle \{x{:}T \mid c\}, e \rangle\!\rangle^{\ell} : \{x{:}T \mid c\}}\ \text{T\_WCHECK}$$

$$\frac{\Sigma;\gamma \vdash \Gamma \quad \Sigma;\gamma;\emptyset \vdash \{x{:}T \mid c\} \quad \Sigma;\gamma;\emptyset \vdash v : T \quad \emptyset;\Sigma;\gamma \vdash p : \mathsf{bool}^{\emptyset} \quad \emptyset \mid \nu\emptyset.\langle\emptyset \mid [\,v/x\,]\,c\rangle \hookrightarrow^* p}{\Sigma;\gamma;\Gamma \vdash \langle\{x{:}T \mid c\}, p, v\rangle^{\ell} : \{x{:}T \mid c\}}\ \text{T\_ACHECK}$$

$$\frac{\Sigma;\gamma \vdash \Gamma \quad \Sigma;\gamma;\emptyset \vdash \{x{:}T \mid c\} \quad \Sigma;\gamma;\emptyset \vdash v : T \quad \emptyset \models [\,v/x\,]\,c}{\Sigma;\gamma;\Gamma \vdash v : \{x{:}T \mid c\}}\ \text{T\_EXACT}$$

$$\frac{\Sigma;\gamma \vdash \Gamma \quad \Sigma;\gamma;\emptyset \vdash v : \{x{:}T \mid c\}}{\Sigma;\gamma;\Gamma \vdash v : T}\ \text{T\_FORGET} \qquad \frac{\Sigma;\gamma \vdash \Gamma \quad \Sigma;\gamma;\emptyset \vdash e : T_1 \quad T_1 \equiv T_2 \quad \Sigma;\gamma;\emptyset \vdash T_2}{\Sigma;\gamma;\Gamma \vdash e : T_2}\ \text{T\_CONV}$$

$\boxed{\mu;\Sigma;\gamma;\Gamma \vdash c : \{A_1\}x{:}T\{A_2\}^{\varrho}}$ **Run-time Computation Typing Rules (Selected)**

$$\frac{\Sigma;\gamma \vdash \Gamma \quad \Sigma;\gamma;\emptyset \vdash \{A_1\}x{:}T\{A_2\}^{\varrho}}{\mu;\Sigma;\gamma;\Gamma \vdash \Uparrow\ell : \{A_1\}x{:}T\{A_2\}^{\varrho}}\ \text{CT\_BLAME} \qquad \frac{\begin{array}{c}\Sigma;\gamma \vdash \Gamma \quad \mu;\Sigma;\gamma;\emptyset \vdash c_1 : \{A_1\}y{:}T_1\{A_3\}^{\varrho_1} \\ \emptyset;\Sigma;\gamma;y{:}T_1 \vdash c_2 : \{A_3\}x{:}T_2\{A_2\}^{\varrho_2} \quad \Sigma;\gamma;\emptyset \vdash \{A_1\}x{:}T_2\{A_3\}^{\varrho_1 \cup \varrho_2}\end{array}}{\mu;\Sigma;\gamma;\Gamma \vdash y \leftarrow \mathsf{do}\ c_1; c_2 : \{A_1\}x{:}T_2\{A_2\}^{\varrho_1 \cup \varrho_2}}\ \text{CT\_CBIND}$$

$$\frac{\Sigma;\gamma \vdash \Gamma \quad \emptyset;\Sigma;\gamma;\emptyset \vdash c_3 : \{A_1, c_1\}x{:}T\{A_2\}^{\langle\gamma_{\mathsf{r}},\gamma_{\mathsf{w}}\rangle} \quad \mu;\Sigma;\gamma \vdash p_2 : \mathsf{bool}^{\gamma_{\mathsf{r}}} \quad \mu \mid \nu\emptyset.\langle\emptyset \mid c_1\rangle \hookrightarrow^* p_2}{\mu;\Sigma;\gamma;\Gamma \vdash \langle \mathsf{assert}\,(c_1), p_2\rangle^{\ell}; c_3 : \{A_1\}x{:}T\{A_2\}^{\langle\gamma_{\mathsf{r}},\gamma_{\mathsf{w}}\rangle}}\ \text{CT\_CHECK}$$

$$\frac{\Sigma;\gamma \vdash \Gamma \quad \mu;\Sigma;\gamma;\emptyset \vdash c : \{A_{11}\}x{:}T_1\{A_{12}\}^{\varrho} \quad \{A_{11}\}x{:}T_1\{A_{12}\}^{\varrho} \equiv \{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho} \quad \Sigma;\gamma;\emptyset \vdash \{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho}}{\mu;\Sigma;\gamma;\Gamma \vdash c : \{A_{21}\}x{:}T_2\{A_{22}\}^{\varrho}}\ \text{CT\_CONV}$$

$\boxed{\mu;\Sigma;\gamma \vdash p : T^{\gamma'}}$ $\boxed{\gamma \vdash \mu : \Sigma^{\gamma'}}$ **Checking State Typing and Store Well-Formedness Rules**

$$\frac{\gamma,\gamma' \vdash \mu' : (\Sigma,\Sigma')^{\gamma'} \quad dom\,(\mu') = dom\,(\Sigma') \quad \mu \uplus \mu';\Sigma,\Sigma';\gamma,\gamma';\emptyset \vdash c' : \{A_1\}x{:}T\{\top\}^{\langle\gamma'\cup\gamma'',\gamma'\rangle} \quad \mu \uplus \mu' \models A_1}{\mu;\Sigma;\gamma \vdash \nu\gamma'.\langle\mu' \mid c'\rangle : T^{\gamma''}}\ \text{PT}$$

$$\frac{dom\,(\mu) = dom\,(\Sigma|_{\gamma'}) \quad \forall\, a@r \in dom\,(\mu).\, \Sigma;\gamma;\emptyset \vdash \mu(a@r) : \Sigma(a@r)}{\gamma \vdash \mu : \Sigma^{\gamma'}}\ \text{WF\_STORE}$$

Figure 6. Run-time typing rules.

As shown in Figure 6, the run-time type system consists of extensions of judgments in Section 4 with store typing contexts—in addition, the computation typing judgment $\mu;\Sigma;\gamma;\Gamma \vdash c : \{A_1\}x{:}T\{A_2\}^{\varrho}$ refers to μ denoting current stores—and two new judgments for checking states $\mu;\Sigma;\gamma \vdash p : T^{\gamma'}$, which means that the computation of p may refer to memory cells at γ' and returns a value of T (if any) under the store μ together with the local store of p, and for stores $\gamma \vdash \mu : \Sigma^{\gamma'}$, which means that all memory cells in store μ are allocated at γ' and their contents are assigned types by Σ. The computation and checking state typing judgments need a current store μ for simulating contract checks in the type system; see (CT_CHECK) below for details. In what follows, we write $\mu \models A$ when, for any $c \in A$, $\mu \mid \nu\emptyset.\langle\emptyset \mid c\rangle \hookrightarrow^* \nu\gamma'.\langle\mu' \mid \mathsf{return}\ \mathsf{true}\rangle$.

Figure 6 shows selected rules for well-formedness and typing judgments. The rules for typing context well-formedness, type well-formedness, and assertion well-formedness are similar to the rules in Figure 3 except for the use of store typing contexts. Since contracts are specifications, refinements and pre- and postconditions should not depend on a current store, so they must be well typed under the empty store.

There are several additional typing rules for run-time terms. In these typing rules, typing contexts in their premises are empty since run-time terms are closed; however, the conclusions allow nonempty typing contexts because run-time terms can be put under binders by substitution in (T_APP). The first five typing rules are syntax-directed. The rule (T_GUARD) requires the contents types of a reference guard to be compatible since the reference guard uses casts between these types when dereference and assignment are applied. Exceptions can be typed at any well-formed type by (T_BLAME)—it is important for showing preservation. The rule (T_ACHECK) requires p in an active check to return Boolean values (if any) in the fourth premise and to be an intermediate state during the check of c in the last premise. The rule (T_EXACT) allows values of T satisfying a contract c to be typed at the refinement type $\{x{:}T \mid c\}$. By the rule (T_FORGET), which corresponds with (R_FORGET), we can peel off the outermost contract of a refinement type. The final rule (T_CONV) is introduced by Belo et al. [3] to show subject reduction in manifest contracts with dependent function types. To see its motivation, let us consider well typed term application $v_1\ e_2$. From (T_APP), the type of $v_1\ e_2$ would be $[\,e_2/x\,]\,T_2$ for some x and T_2. If e_2 reduces to term e_2', the type of the application changes to $[\,e_2'/x\,]\,T_2$, which is different from $[\,e_2/x\,]\,T_2$ in general. Thus, subject reduction would not hold if there are no ways to connect $[\,e_2/x\,]\,T$ with $[\,e_2'/x\,]\,T$. In fact, the rule (T_CONV) does connect these two types by allowing terms to be retyped at different, but equivalent types. The type equivalence, denoted by $\equiv$, is given as follows:

Definition 1 (Type Equivalence)**.** *$T_1 \Rrightarrow T_2$ iff there exist some T, x, E, e_1, and e_2 such that $T_1 = [\,E[e_1]/x\,]\,T$, $T_2 = [\,E[e_2]/x\,]\,T$, and $e_1 \rightsquigarrow e_2$. Type equivalence $\equiv$ is the symmetric and transitive closure of $\Rrightarrow$.*

Computation typing rules are also added. The rule (CT_CBIND), which looks similar to (CT_BIND), accepts bind constructs $x \leftarrow \mathsf{do}\ c_1; c_2$ where c_1 and c_2 are typed under the current store μ and the empty store, respectively; the differences of stores in c_1 and c_2 stems from the fact that c_1 will be executed under μ but c_2 may or may not (since c_1 can mutate the store).

Similarly, remaining computations in other rules are also required to be typed under the empty store. The rule (CT_CHECK), applied to assertion checks, requires the checking state p to be well typed under the current store μ in the third premise and be an actual intermediate state of checking c_1 in the last premise. The final rule (CT_CONV) is needed because do-expressions can be typed at equivalent types by (T_CONV).

Store well-formedness is derived by (WF_STORE), which demands that all memory cells in well-formed stores be allocated at given regions γ' and their contents have types given by store typing contexts; $\Sigma|_{\gamma'}$ is the same store typing context as Σ except that its domain is restricted to addresses with regions in γ'.

Finally, using a local store typing context Σ', the rule (PT) gives type T to checking state $\nu\gamma'.\langle\mu' \mid c'\rangle$ if and only if the followings hold. First, the local store μ' maps only references at the local regions γ' and each value in it has the type assigned by Σ'. Second, the domains of μ' and Σ' coincide. Third, using the local information μ', Σ', and γ', computation c' produces values of T (if any) with read-only references at regions γ'' as well as unrestricted references at local regions γ'. Finally, the precondition of c' holds under the concatenation of the program store and the local store.

6.2 Type Soundness

We show type soundness, which says that, given a well typed program c of $\{\top\}x{:}T\{A_2\}^{\langle\{r\},\{r\}\rangle}$, $p = \nu\emptyset.\langle\emptyset \mid \nu r.\, c\rangle$ results in a value, raises an exception, or diverges and, moreover, if p terminates at value v with store μ, v satisfies refinements in T and $\mu \models [\,v/x\,]\,A_2$ holds. Although the proof of type soundness follows progress and preservation as in the previous work [48], this paper states just their simplified versions; the full statements are shown in the supplementary material. We assume that $ty\,(op)$ and $[\![op]\!]$ agree in a certain sense; also see the supplementary material for details.

Lemma 2 (Value Inversion)**.** *If $\Sigma;\gamma;\emptyset \vdash v : \{x{:}T \mid c\}$, then $\emptyset \models [\,v/x\,]\,c$.*

Lemma 3 (Progress)**.** *If $\emptyset;\Sigma;\gamma \vdash p : T^{\emptyset}$, then: (1) $\emptyset \mid p \hookrightarrow p'$ for some p'; (2) $p = \nu\gamma'.\langle\mu' \mid \mathsf{return}\ v'\rangle$ for some γ', μ', and v'; or (3) $p = \nu\gamma'.\langle\mu' \mid \Uparrow\ell'\rangle$ for some γ', μ', and ℓ'.*

Lemma 4 (Preservation)**.** *If $\emptyset;\Sigma;\gamma \vdash p : T^{\emptyset}$ and $\emptyset \mid p \hookrightarrow p'$, then $\emptyset;\Sigma;\gamma \vdash p' : T^{\emptyset}$.*

Lemma 5 (Postcondition Satisfaction)**.** *If (1) $\mu;\Sigma;\gamma;\emptyset \vdash c : \{A_1\}x{:}T\{A_2\}^{\langle\gamma,\gamma\rangle}$, (2) $\gamma \vdash \mu : \Sigma^{\gamma}$, (3) $\mu \models A_1$, and (4) $\emptyset \mid \nu\gamma.\langle\mu \mid c\rangle \hookrightarrow^* \nu\gamma'.\langle\mu' \mid \mathsf{return}\ v'\rangle$, then $\mu' \models [\,v'/x\,]\,A_2$.*

Theorem 1 (Type Soundness)**.** *Suppose that $\emptyset;\emptyset;\{r\};\emptyset \vdash c : \{\top\}x{:}T\{A_2\}^{\langle\{r\},\{r\}\rangle}$. Let $p = \nu\emptyset.\langle\emptyset \mid \nu r.\, c\rangle$. Then, one of the followings hold: (1) $\emptyset \mid p \hookrightarrow^* \nu\gamma.\langle\mu \mid \mathsf{return}\ v\rangle$ for some γ, μ, and v; (2) $\emptyset \mid p \hookrightarrow^* \nu\gamma.\langle\mu \mid \Uparrow\ell\rangle$ for some γ, μ, and ℓ; or (3) there is an infinite sequence of computation $\emptyset \mid p \hookrightarrow p_1, \emptyset \mid p_1 \hookrightarrow p_2, \cdots$. Moreover, if (1) holds, then: (a) if $T = \{x{:}T_0 \mid c_0\}$, then $\emptyset \models [\,v/x\,]\,c_0$; and (b) $\mu \models [\,v/x\,]\,A_2$.*

Proof. By Progress and Preservation. The properties (a) and (b) are shown by Lemmas 2 and 5. □

7. Static Contract Verification

This work studies "post facto" static verification of state-dependent contracts—more precisely, we identify assertions such that programs with and without them are contextually equivalent [3]. This paper focuses on two verification techniques: elimination of assertions for pre- and postconditions and region-based local reasoning. Note that, although we are interested in, this paper is not concerned about specific verification algorithms; instead, we study *what* state-dependent contracts static checking can verify as in the earlier work on manifest contracts [3, 18, 30]. Static verification of state-independent contracts in terms of elimination of casts is left for future work, although we expect it is not very different from previous work. We do not present the formal definition of contextual equivalence here; interested readers are referred to the supplementary material.

7.1 Elimination of Pre- and Postcondition Assertions

An assertion is redundant and can be eliminated if the assertion is implied by the contracts established by a preceding computation. For example, consider string table t and string s that satisfy $y \leftarrow \mathtt{mem}\ t\ s; \mathsf{return}\ y$. Then, asserting $x \leftarrow \mathtt{size}\ t; \mathsf{return}\ x > 0$ (`size` is a function to return the size of a given table) should be redundant. In fact, we could prove that $y \leftarrow \mathtt{mem}\ t\ s; \mathsf{return}\ y$ "implies" $x \leftarrow \mathtt{size}\ t; \mathsf{return}\ x > 0$—whenever the former returns `true`, the latter returns `true`—if we know the concrete implementation type of string tables. To formalize this notion of implication, we consider closing substitutions, which give interpretations to free variables, and possible stores under which contracts evaluate. In what follows, σ denotes mappings from term variables to values and from region variables to region variables and write $\sigma(\gamma)$ for the image of γ under σ.

Definition 2 (Closing Substitution and Possible Store)**.** *We write $\Sigma;\gamma;\Gamma \vdash \langle\mu,\sigma\rangle^{\langle\gamma_{\mathtt{r}},\gamma_{\mathtt{w}}\rangle}$ when there exist some Σ' and γ' such that: (1) $\Sigma \subseteq \Sigma'$; (2) $\gamma \subseteq \gamma'$; (3) for any $r \in \gamma$, $r \notin dom\,(\sigma)$; (4) for any $r \in \Gamma$, $\sigma(r) \in \gamma'$; (5) for any $x{:}T \in \Gamma$, $\Sigma';\gamma';\emptyset \vdash \sigma(x) : \sigma(T)$; and (6) $\gamma' \vdash \mu : \Sigma'^{\sigma(\gamma_{\mathtt{r}})\cup\sigma(\gamma_{\mathtt{w}})}$. We write $\Sigma;\gamma;\Gamma \vdash \sigma$ for $\Sigma;\gamma;\Gamma \vdash \langle\emptyset,\sigma\rangle^{\langle\emptyset,\emptyset\rangle}$.*

What is meant by "contract A' is implied from A" is that, for any interpretation of free variables and store such that A results in `true`, so does A'. We write A, A' for the concatenation of A and A'.

Definition 3 (Contract Implication)**.** *Suppose that $\gamma;\Gamma \vdash^{\varrho} A, A'$. Then, A'* is implied from *A if, for any μ and σ such that $\emptyset;\gamma;\Gamma \vdash \langle\mu,\sigma\rangle^{\varrho}$ and $\mu \models \sigma(A)$, $\mu \models \sigma(A')$ holds.*

Lemma 6 (Precondition Assertion Elimination)**.** *Suppose that $\gamma;\Gamma \vdash c : \{A_1, c_1\}x{:}T\{A_2\}^{\varrho}$. If c_1 is implied from A_1, $\mathsf{assert}\,(c_1)^{\ell}; c$ and c are contextually equivalent.*

Since postcondition checks are a derived form using assertions, we can show elimination of postcondition assertions as a corollary.

Corollary 1 (Postcondition Assertion Elimination)**.** *Suppose that $\gamma;\Gamma \vdash c : \{A_1\}x{:}T\{A_2\}^{\varrho}$. For any c_2, if $\gamma;\Gamma, x{:}T \vdash^{\varrho} A_2, c_2$ and c_2 is implied from A_2, then $c;\lambda x.\mathsf{assert}\,(c_2)^{\ell}$ and c are contextually equivalent.*

As an application of these elimination lemmas, let us consider a program which, given a string table t, adds a fresh string to t, produces the average length of strings in t, and ensures that the average is not negative. Such a program can be given as follows:

$$\begin{array}{l}\lambda t{:}\mathsf{tbl}.\ s \leftarrow \mathtt{fresh\_str}\ t; \_ \leftarrow \mathtt{add}\ t\ s;\\ \quad \mathsf{assert}\,(x \leftarrow \mathtt{size}\ t; \mathsf{return}\ x > 0)^{\ell_1};\\ \quad a \leftarrow \mathtt{average\_length}\ t;\\ \quad \mathsf{return}\ a; \lambda a.\mathsf{assert}\,(a \geq 0)^{\ell_2}\end{array}$$

where `average_length` calculates the average length of strings in an argument table. The type of the program is:

$$t{:}\mathsf{tbl} \to \{\top\}a{:}\mathsf{float}\{\mathsf{return}\ a \geq 0\}^{\langle\{r\},\emptyset\rangle},$$

where r is a region at which argument tables are allocated, and the type of `average_length` is:

$$\begin{array}{l} t{:}\mathsf{tbl} \to \{x \leftarrow \mathtt{size}\, t; \mathsf{return}\, x > 0\} \\ \quad a{:}\mathsf{float} \\ \quad \{y \leftarrow \mathtt{exists}\, (\lambda s{:}\mathtt{string}.\mathtt{length}\, s \geq a)\, t; \\ \quad\ z \leftarrow \mathtt{exists}\, (\lambda s{:}\mathtt{string}.a \geq \mathtt{length}\, s)\, t; \\ \quad \mathsf{return}\, y \,\&\, z\}^{\langle\{r\},\emptyset\rangle} \end{array}$$

(`exists` returns whether a given table has a string satisfying a given predicate), where it requires a given table to have at least one element and guarantees that the average is equal to or less than the length of the longest string and is equal to or more than the length of the shortest one. The program has two run-time checks: one is for checking the precondition of `average_length` and the other is for ensuring the postcondition of the program. If the concrete implementation of functions used in the program is known, we could prove that the precondition of `average_length` is implied from the postcondition of `add` (that is, $y \leftarrow \mathtt{mem}\, t\, s; \mathsf{return}\, y$) and the postcondition of the program is from that of `average_length` (because the length of any string is equal to or more than 0). Thus, using Lemma 6 and Corollary 1, we can show that the program above is contextually equivalent to:

$$\begin{array}{l} \lambda t{:}\mathsf{tbl}.\ s \leftarrow \mathtt{fresh\_str}\, t; \_ \leftarrow \mathtt{add}\, t\, s; \\ \quad a \leftarrow \mathtt{average\_length}\, t; \\ \quad \mathsf{return}\, a \end{array}$$

which have no run-time checks!

These elimination techniques also enable us to strengthen preconditions and weaken postconditions semantically.

Corollary 2 (Semantic Weakening)**.** *Suppose that* $\gamma; \Gamma \vdash c : \{A_1\}x{:}T\{A_2\}^{\varrho}$. *For any* A_1' *and* A_2', *if* $\gamma; \Gamma \vdash^{\varrho} A_1'$ *and* $\gamma; \Gamma, x{:}T \vdash^{\varrho} A_2'$ *and* A_1 *is implied from* A_1' *and* A_2' *is implied from* A_2, *then* $\mathsf{assert}\,(A_1)^{\ell_1}; c; \lambda x.\mathsf{assert}\,(A_2')^{\ell_2}$ *and* c *are contextually equivalent at* $\{A_1'\}x{:}T\{A_2'\}^{\varrho}$.

7.2 Region-Based Local Reasoning

Local reasoning allows applying verification methods to subcomponents of a program locally, so it is important to for verification to scale up to large programs. We achieve *region-based* local reasoning in the form similar to Separation logic [41, 45], where the so-called frame rule plays an important role: given a computation requiring precondition P, which specifies a heap before the computation, and guaranteeing postcondition Q, which specifies the heap after the computation, the rule allows the computation to require a condition R separated from P and Q and guarantee the same condition. It is represented in the form of Hoare triples as follows:

$$\frac{\{P\}\, c\, \{Q\}}{\{P * R\}\, c\, \{Q * R\}}$$

where $P * R$ is the separating conjunction of P and R and states that P and R specify disjoint heaps. In our context, computation c can preserve any condition A if A never mentions references mutated by c. The effect system in Section 4 is useful to verity whether A mentions such references because it can analyze what references computations manipulate with the help of regions. To formalize the local reasoning, we first introduce the notion of "disjointness" of regions—region sets γ_1 and γ_2 are disjoint if the results of application of any region substitutions are disjoint.

Definition 4 (Disjoint Regions)**.** *We write* $\Sigma; \gamma; \Gamma \vdash \gamma_1\, \mathsf{disj}\, \gamma_2$ *when, for any* σ *such that* $\Sigma; \gamma; \Gamma \vdash \sigma$, $\sigma(\gamma_1) \cap \sigma(\gamma_2) = \emptyset$.

Lemma 7 (Local Reasoning)**.** *Suppose* $\gamma; \Gamma \vdash c : \{A_1\}x{:}T\{A_2\}^{\langle\gamma_{\mathtt{r}},\gamma_{\mathtt{w}}\rangle}$. *For any* A *and* $\gamma_{\mathtt{r}}' \subseteq \gamma_{\mathtt{r}}$, *if* $\gamma; \Gamma \vdash^{\langle\gamma_{\mathtt{r}}',\emptyset\rangle} A$ *and* $\emptyset; \gamma; \Gamma \vdash \gamma_{\mathtt{r}}'\, \mathsf{disj}\, \gamma_{\mathtt{w}}$, *then, for any fresh* y, $c; \lambda y.\mathsf{assert}\,(A)^{\ell}$ *and* c *are contextually equivalent at* $\{A_1, A\}x{:}T\{A_2, A\}^{\langle\gamma_{\mathtt{r}},\gamma_{\mathtt{w}}\rangle}$

The local reasoning enables us to recover contract information lost by assignment.

Corollary 3. *Suppose that (1)* $\gamma; \Gamma \vdash e_1 : \mathsf{Ref}_r\, T'$; *(2)* $\gamma; \Gamma \vdash e_2 : T'$; *(3)* $\gamma; \Gamma \vdash^{\langle\gamma_{\mathtt{r}},\emptyset\rangle} A$; *and (4)* $\emptyset; \gamma; \Gamma \vdash \gamma_{\mathtt{r}}\, \mathsf{disj}\, \{r\}$. *Then,* $x \Leftarrow e_1 := e_2; \mathsf{return}\, (); \lambda x.\mathsf{assert}\,(A)^{\ell}$ *and* $x \Leftarrow e_1 := e_2; \mathsf{return}\, ()$ *are contextually equivalent at* $\{A\}x{:}T\{A\}^{\langle\gamma_{\mathtt{r}},\{r\}\rangle}$.

Although the local reasoning would be a useful technique, its applicability rests on how many regions are judged to be disjoint. Unfortunately, $\lambda^{\mathsf{H}}_{\mathsf{ref}}$ does not have a very strong power for this judgment—nonempty region sets are disjoint only if they contain no region variables introduced by region abstraction, which is not very satisfactory because region abstractions are a key feature to promote program reuse and so would be often used. We could address this issue by adding operations on region variables. For example, consider an extension of $\lambda^{\mathsf{H}}_{\mathsf{ref}}$ with an equality operator $r == s$ on regions, which behaves as follows:

$$r == r \rightsquigarrow \mathsf{true} \qquad r == s \rightsquigarrow \mathsf{false} \quad (\text{if } r \neq s)$$

Using this equality, we can state that region r is different from s as a contract and then $\{r\}$ and $\{s\}$ are judged to be disjoint even if either or both of them are abstracted regions because closing substitutions must respect the contract.

8. Related Work

Hoare Type Theory Hoare Type Theory [37, 38] is a theoretical framework to verify stateful programs with higher-order functions statically, incorporating the core ideas of Hoare logic (and Separation logic) into a type system with dependent types. The key idea of HTT is to introduce Hoare types, where pre- and postconditions are written in classical multi-sorted first-order logic with predicates to specify heaps. Computational Hoare types in this work are a variant of HTT's Hoare types and allow pre- and postconditions to be computational so that their dynamic checks are possible. On one hand, in addition to dynamic checking, executable pre- and postconditions enable programmers to give natural specifications using program functions they define, as the last example in Section 2.3, which uses interface functions to mention internal states of an abstract type. More generally, we can reuse *any* program components—for example, even partial functions (their use makes static verification difficult, though)—in specifications easily. Although Nanevski et al. extended HTT to deal with internal states of functions [39], they do not allow interface program functions to mention internal states in specifications—in their work, stateful predicate functions have to take heaps as an argument but program functions do not usually. On the other hand, the expressive power of specifications is restricted—for example, unlike HTT, it appears to be difficult to work well with existential quantifier and specify the relationship between stores before and after computation—though expressive powers of computational and noncomputational Hoare types are incomparable generally because the former accepts nonterminating contracts whereas the latter does not. It is left as future work to give a remedy for the defect of computational Hoare types.

Other work on static verification of stateful programs Other than HTT, there is much research on static verification of stateful programs with dependent type systems. Dependent ML [69, 70] and Applied Type System [68] are programming languages with support for static verification of stateful programs. Specifications in these languages are neither state-dependent nor computational. Vekris et al. [63] study a refinement type system for static verification of TypeScript [34] programs, which are functional, object-oriented, and imperative. Their system supports imperative features, such as variable assignment and objects with mutable fields, from TypeScript and is able to specify not only refinements but

also class invariants. Although specifications in the system can refer to variables and object fields, Vekris et al. deal with only state-*in*dependent specifications by transforming code with variable assignment to a static single assignment form [1, 47] and restricting fields accessible from specifications to immutable ones (field declarations are annotated with immutability). Gordon et al. [20] proposed *rely–guarantee references*, where a reference is augmented with a guarantee relation, which describes possible actions through the reference, and a rely relation, which describes possible actions through aliases, and developed a framework with rely–guarantee references to verify that an assignment to a reference does not invalidate predicates with respect to aliases of the reference. Applying their approach to dynamic checking is interesting, but it would need a run-time mechanism to monitor guarantee and rely relations. Swamy et al. [56, 57] developed Dijkstra monads, a variant of Nanevski et al.'s Hoare types, to verify effectful programs automatically. We expect that their technique can be applied to our work for automatic verification.

In objected-oriented languages, many techniques—e.g., ownership types [10–12], variants of Separation logic [4, 43, 52], dynamic frames [27, 28], implicit dynamic frames [51], regional logic [2], etc.—have been studied to address the frame problem [7], which is a common theme in verification of programs with pointer aliasing. We address the frame problem with Hoare types and a region-based effect system, but the earlier work above would inspire us to refine our approach and, furthermore, to investigate better approaches.

Contracts for references Flanagan, Freund, and Tomb [19] studied combination of static and dynamic approaches to checking contracts of (im)mutable objects. Although their goal is similar to ours, they allow only pure contracts which never depend on even locally allocated references whereas we accept even state-dependent contracts. That work also proposed reference guards, which have been a usual approach to dynamic checking for references [13, 24, 55] (there is also another method [50], though). Perhaps, one might consider that state-dependent contracts could be embedded into reference guards, that is, properties with respect to references could be represented by refining contents of reference types, as type `tbl` in Figure 1. Unfortunately, this approach is not satisfactory because it would not be possible to relate results of two or more stateful computations (e.g., the contract of `add` would be disallowed) or to abstract implementation types of mutable data structures as in the last example of Section 2.3.

Tob and Pucella [61] integrated programs in two languages—one has a conventional type system and the other has an affine type system—by using stateful contracts with assignment. Their system uses contracts to monitor that conventional programs use affine values just once and does not use them as specifications of program components. Disney, Flanagan, and McCarthy [14] proposed a higher-order temporal contract system to monitor temporal behavior of stateful programs by specifying orders in which functions of modules should be called. Though they dealt with predicate contracts with imperative features, the issue of state-dependent contracts is not in their interests.

9. Conclusion

We address the issue of state-dependent contracts in hybrid contract checking based on manifest contract systems by introducing a region-based effect system with computational Hoare types. To formalize our ideas, we define $\lambda^{\mathrm{H}}_{\mathsf{ref}}$, where refinements are checked with casts and pre- and postconditions of Hoare types are checked with assertions, and show its type soundness. We also study "post facto" static verification, in particular, elimination of assertions for pre- and postconditions and region-based local reasoning. This work is a stepping stone for integrating static and dynamic verification of state-dependent contracts and we have many directions of future work. For example, it is interesting to investigate how we can strengthen our contract language so that relationships between heaps before and after computation can be expressed. Implementation of our calculus is also left for future work.

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