Nat

Syntax:

$$\boldsymbol{n} \in \mathbf{Nat} ::= \mathtt{Z} \mid \mathtt{S}(\boldsymbol{n})$$

Judgment Form(s):

$$\boldsymbol{n_1}\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}$$

$$\boldsymbol{n_1}\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}$$

Derivation Rules:

$$\frac{}{\mathtt{Z}\ \mathtt{plus}\ \boldsymbol{n}\ \mathtt{is}\ \boldsymbol{n}} \quad \text{(P-Zero)}$$

$$\frac{\boldsymbol{n_1}\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n}}{\mathtt{S}(\boldsymbol{n_1})\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \mathtt{S}(\boldsymbol{n})} \quad \text{(P-Succ)}$$

$$\frac{}{\mathtt{Z}\ \mathtt{times}\ \boldsymbol{n}\ \mathtt{is}\ \mathtt{Z}} \quad \text{(T-Zero)}$$

$$\frac{\boldsymbol{n_1}\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3} \qquad \boldsymbol{n_2}\ \mathtt{plus}\ \boldsymbol{n_3}\ \mathtt{is}\ \boldsymbol{n_4}}{\mathtt{S}(\boldsymbol{n_1})\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_4}} \quad \text{(T-Succ)}$$

CompareNat1

Syntax:

$$\boldsymbol{n} \in \mathbf{Nat} ::= \texttt{Z} \mid \texttt{S}(\boldsymbol{n})$$

Judgment Form(s):

$$\boldsymbol{n_1} \texttt{ is less than } \boldsymbol{n_2}$$

Derivation Rules:

$$\frac{}{\boldsymbol{n} \texttt{ is less than S}(\boldsymbol{n})} \quad \text{(L-Succ)}$$

$$\frac{\boldsymbol{n_1} \texttt{ is less than } \boldsymbol{n_2} \qquad \boldsymbol{n_2} \texttt{ is less than } \boldsymbol{n_3}}{\boldsymbol{n_1} \texttt{ is less than } \boldsymbol{n_3}} \quad \text{(L-Trans)}$$

CompareNat2

Syntax:

$$\boldsymbol{n} \in \mathbf{Nat} ::= \texttt{Z} \mid \texttt{S}(\boldsymbol{n})$$

Judgment Form(s):

$$\boldsymbol{n_1} \texttt{ is less than } \boldsymbol{n_2}$$

Derivation Rules:

$$\frac{}{\texttt{Z is less than S}(\boldsymbol{n})} \quad \text{(L-Zero)}$$

$$\frac{\boldsymbol{n_1} \texttt{ is less than } \boldsymbol{n_2}}{\texttt{S}(\boldsymbol{n_1}) \texttt{ is less than S}(\boldsymbol{n_2})} \quad \text{(L-SuccSucc)}$$

CompareNat3

Syntax:

$$\boldsymbol{n} \in \mathbf{Nat} ::= \texttt{Z} \mid \texttt{S}(\boldsymbol{n})$$

Judgment Form(s):

$$\boldsymbol{n_1} \texttt{ is less than } \boldsymbol{n_2}$$

Derivation Rules:

$$\frac{}{\boldsymbol{n} \texttt{ is less than S}(\boldsymbol{n})} \quad \text{(L-Succ)}$$

$$\frac{\boldsymbol{n_1} \texttt{ is less than } \boldsymbol{n_2}}{\boldsymbol{n_1} \texttt{ is less than S}(\boldsymbol{n_2})} \quad \text{(L-SuccR)}$$

EvalNatExp

Syntax:

$$\begin{aligned} \boldsymbol{n} \in \mathbf{Nat} &::= \mathtt{Z} \mid \mathtt{S}(\boldsymbol{n}) \\ \boldsymbol{e} \in \mathbf{Exp} &::= \boldsymbol{n} \mid \boldsymbol{e} + \boldsymbol{e} \mid \boldsymbol{e} * \boldsymbol{e} \end{aligned}$$

Judgment Form(s):

$$\boldsymbol{e} \Downarrow \boldsymbol{n}$$

$$\boldsymbol{n_1}\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}$$

$$\boldsymbol{n_1}\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}$$

Derivation Rules:

$$\frac{}{\boldsymbol{n} \Downarrow \boldsymbol{n}} \quad \text{(E-Const)}$$

$$\frac{\boldsymbol{e_1} \Downarrow \boldsymbol{n_1} \qquad \boldsymbol{e_2} \Downarrow \boldsymbol{n_2} \qquad \boldsymbol{n_1}\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n}}{\boldsymbol{e_1} + \boldsymbol{e_2} \Downarrow \boldsymbol{n}} \quad \text{(E-Plus)}$$

$$\frac{\boldsymbol{e_1} \Downarrow \boldsymbol{n_1} \qquad \boldsymbol{e_2} \Downarrow \boldsymbol{n_2} \qquad \boldsymbol{n_1}\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n}}{\boldsymbol{e_1} * \boldsymbol{e_2} \Downarrow \boldsymbol{n}} \quad \text{(E-Times)}$$

$$\frac{}{\mathtt{Z}\ \mathtt{plus}\ \boldsymbol{n}\ \mathtt{is}\ \boldsymbol{n}} \quad \text{(P-Zero)}$$

$$\frac{\boldsymbol{n_1}\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n}}{\mathtt{S}(\boldsymbol{n_1})\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \mathtt{S}(\boldsymbol{n})} \quad \text{(P-Succ)}$$

$$\frac{}{\mathtt{Z}\ \mathtt{times}\ \boldsymbol{n}\ \mathtt{is}\ \mathtt{Z}} \quad \text{(T-Zero)}$$

$$\frac{\boldsymbol{n_1}\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3} \qquad \boldsymbol{n_2}\ \mathtt{plus}\ \boldsymbol{n_3}\ \mathtt{is}\ \boldsymbol{n_4}}{\mathtt{S}(\boldsymbol{n_1})\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_4}} \quad \text{(T-Succ)}$$

ReduceNatExp

Syntax:

$$\begin{array}{l} \boldsymbol{n} \in \mathbf{Nat} ::= \mathtt{Z} \mid \mathtt{S}(\boldsymbol{n}) \\ \boldsymbol{e} \in \mathbf{Exp} ::= \boldsymbol{n} \mid \boldsymbol{e} + \boldsymbol{e} \mid \boldsymbol{e} * \boldsymbol{e} \end{array}$$

Judgment Form(s):

$$\boldsymbol{e_1} \longrightarrow \boldsymbol{e_2}$$

$$\boldsymbol{e_1} \longrightarrow_d \boldsymbol{e_2}$$

$$\boldsymbol{e_1} \longrightarrow^* \boldsymbol{e_2}$$

$$\boldsymbol{n_1}\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}$$

$$\boldsymbol{n_1}\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}$$

Derivation Rules:

$$\frac{\boldsymbol{n_1}\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}}{\boldsymbol{n_1} + \boldsymbol{n_2} \longrightarrow \boldsymbol{n_3}} \quad \text{(R-Plus)}$$

$$\frac{\boldsymbol{n_1}\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}}{\boldsymbol{n_1} * \boldsymbol{n_2} \longrightarrow \boldsymbol{n_3}} \quad \text{(R-Times)}$$

$$\frac{\boldsymbol{e_1} \longrightarrow \boldsymbol{e_1'}}{\boldsymbol{e_1} + \boldsymbol{e_2} \longrightarrow \boldsymbol{e_1'} + \boldsymbol{e_2}} \quad \text{(R-PlusL)}$$

$$\frac{\boldsymbol{e_2} \longrightarrow \boldsymbol{e_2'}}{\boldsymbol{e_1} + \boldsymbol{e_2} \longrightarrow \boldsymbol{e_1} + \boldsymbol{e_2'}} \quad \text{(R-PlusR)}$$

$$\frac{\boldsymbol{e_1} \longrightarrow \boldsymbol{e_1'}}{\boldsymbol{e_1} * \boldsymbol{e_2} \longrightarrow \boldsymbol{e_1'} * \boldsymbol{e_2}} \quad \text{(R-TimesL)}$$

$$\frac{\boldsymbol{e_2} \longrightarrow \boldsymbol{e_2'}}{\boldsymbol{e_1} * \boldsymbol{e_2} \longrightarrow \boldsymbol{e_1} * \boldsymbol{e_2'}} \quad \text{(R-TimesR)}$$

$$\frac{\boldsymbol{n_1}\ \mathtt{plus}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}}{\boldsymbol{n_1} + \boldsymbol{n_2} \longrightarrow_d \boldsymbol{n_3}} \quad \text{(DR-Plus)}$$

$$\frac{\boldsymbol{n_1}\ \mathtt{times}\ \boldsymbol{n_2}\ \mathtt{is}\ \boldsymbol{n_3}}{\boldsymbol{n_1} * \boldsymbol{n_2} \longrightarrow_d \boldsymbol{n_3}} \quad \text{(DR-Times)}$$

$$\frac{e_1 \longrightarrow_d e_1'}{e_1 + e_2 \longrightarrow_d e_1' + e_2} \quad \text{(DR-PlusL)}$$

$$\frac{e_2 \longrightarrow_d e_2'}{n_1 + e_2 \longrightarrow_d n_1 + e_2'} \quad \text{(DR-PlusR)}$$

$$\frac{e_1 \longrightarrow_d e_1'}{e_1 * e_2 \longrightarrow_d e_1' * e_2} \quad \text{(DR-TimesL)}$$

$$\frac{e_2 \longrightarrow_d e_2'}{n_1 * e_2 \longrightarrow_d n_1 * e_2'} \quad \text{(DR-TimesR)}$$

$$\frac{}{e \longrightarrow^* e} \quad \text{(MR-Zero)}$$

$$\frac{e \longrightarrow^* e' \qquad e' \longrightarrow^* e''}{e \longrightarrow^* e''} \quad \text{(MR-Multi)}$$

$$\frac{e \longrightarrow e'}{e \longrightarrow^* e'} \quad \text{(MR-One)}$$

$$\frac{}{\texttt{Z plus } n \texttt{ is } n} \quad \text{(P-Zero)}$$

$$\frac{n_1 \texttt{ plus } n_2 \texttt{ is } n}{\texttt{S(}n_1\texttt{) plus } n_2 \texttt{ is S(}n\texttt{)}} \quad \text{(P-Succ)}$$

$$\frac{}{\texttt{Z times } n \texttt{ is Z}} \quad \text{(T-Zero)}$$

$$\frac{n_1 \texttt{ times } n_2 \texttt{ is } n_3 \qquad n_2 \texttt{ plus } n_3 \texttt{ is } n_4}{\texttt{S(}n_1\texttt{) times } n_2 \texttt{ is } n_4} \quad \text{(T-Succ)}$$

EvalML1

Syntax:

$$
\begin{array}{l}
i \in \textbf{int} \\
b \in \textbf{bool} \\
v \in \textbf{Value} ::= i \mid b \\
e \in \textbf{Exp} ::= i \mid b \mid e\ op\ e \mid \texttt{if}\ e\ \texttt{then}\ e\ \texttt{else}\ e \\
op \in \textbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{array}
$$

Judgment Form(s):

$$e \Downarrow v$$

$$i_1\ \texttt{plus}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{minus}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{times}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{less than}\ i_2\ \texttt{is}\ b_3$$

Derivation Rules:

$$\frac{}{i \Downarrow i} \quad \text{(E-Int)}$$

$$\frac{}{b \Downarrow b} \quad \text{(E-Bool)}$$

$$\frac{e_1 \Downarrow \texttt{true} \qquad e_2 \Downarrow v}{\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 \Downarrow v} \quad \text{(E-IfT)}$$

$$\frac{e_1 \Downarrow \texttt{false} \qquad e_3 \Downarrow v}{\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 \Downarrow v} \quad \text{(E-IfF)}$$

$$\frac{e_1 \Downarrow i_1 \qquad e_2 \Downarrow i_2 \qquad i_1\ \texttt{plus}\ i_2\ \texttt{is}\ i_3}{e_1 + e_2 \Downarrow i_3} \quad \text{(E-Plus)}$$

$$\frac{e_1 \Downarrow i_1 \qquad e_2 \Downarrow i_2 \qquad i_1\ \texttt{minus}\ i_2\ \texttt{is}\ i_3}{e_1 - e_2 \Downarrow i_3} \quad \text{(E-Minus)}$$

$$\frac{e_1 \Downarrow i_1 \qquad e_2 \Downarrow i_2 \qquad i_1\ \texttt{times}\ i_2\ \texttt{is}\ i_3}{e_1 * e_2 \Downarrow i_3} \quad \text{(E-Times)}$$

$$\frac{e_1 \Downarrow i_1 \qquad e_2 \Downarrow i_2 \qquad i_1 \texttt{ less than } i_2 \texttt{ is } b_3}{e_1 \texttt{ < } e_2 \Downarrow b_3} \tag{E-Lt}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1 \texttt{ plus } i_2 \texttt{ is } i_3} \tag{B-Plus}$$

$$\frac{(i_3 = i_1 - i_2)}{i_1 \texttt{ minus } i_2 \texttt{ is } i_3} \tag{B-Minus}$$

$$\frac{(i_3 = i_1 * i_2)}{i_1 \texttt{ times } i_2 \texttt{ is } i_3} \tag{B-Times}$$

$$\frac{(b_3 = (i_1 < i_2))}{i_1 \texttt{ less than } i_2 \texttt{ is } b_3} \tag{B-Lt}$$

EvalML1Err

Syntax:

$$
\begin{array}{l}
\boldsymbol{i} \in \textbf{int} \\
\boldsymbol{b} \in \textbf{bool} \\
\boldsymbol{v} \in \textbf{Value} ::= \boldsymbol{i} \mid \boldsymbol{b} \\
\boldsymbol{r} \in \textbf{Res} ::= \boldsymbol{v} \mid \textbf{error} \\
\boldsymbol{e} \in \textbf{Exp} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid \boldsymbol{e}\ \boldsymbol{op}\ \boldsymbol{e} \mid \texttt{if}\ \boldsymbol{e}\ \texttt{then}\ \boldsymbol{e}\ \texttt{else}\ \boldsymbol{e} \\
\boldsymbol{op} \in \textbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{array}
$$

Judgment Form(s):

$$\boldsymbol{e} \Downarrow \boldsymbol{r}$$

$$\boldsymbol{i_1}\ \texttt{plus}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}$$

$$\boldsymbol{i_1}\ \texttt{minus}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}$$

$$\boldsymbol{i_1}\ \texttt{times}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}$$

$$\boldsymbol{i_1}\ \texttt{less than}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{b_3}$$

Derivation Rules:

$$\frac{}{\boldsymbol{i} \Downarrow \boldsymbol{i}} \quad \text{(E-Int)}$$

$$\frac{}{\boldsymbol{b} \Downarrow \boldsymbol{b}} \quad \text{(E-Bool)}$$

$$\frac{\boldsymbol{e_1} \Downarrow \texttt{true} \qquad \boldsymbol{e_2} \Downarrow \boldsymbol{v}}{\texttt{if}\ \boldsymbol{e_1}\ \texttt{then}\ \boldsymbol{e_2}\ \texttt{else}\ \boldsymbol{e_3} \Downarrow \boldsymbol{v}} \quad \text{(E-IfT)}$$

$$\frac{\boldsymbol{e_1} \Downarrow \texttt{false} \qquad \boldsymbol{e_3} \Downarrow \boldsymbol{v}}{\texttt{if}\ \boldsymbol{e_1}\ \texttt{then}\ \boldsymbol{e_2}\ \texttt{else}\ \boldsymbol{e_3} \Downarrow \boldsymbol{v}} \quad \text{(E-IfF)}$$

$$\frac{\boldsymbol{e_1} \Downarrow \boldsymbol{i_1} \qquad \boldsymbol{e_2} \Downarrow \boldsymbol{i_2} \qquad \boldsymbol{i_1}\ \texttt{plus}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}}{\boldsymbol{e_1} + \boldsymbol{e_2} \Downarrow \boldsymbol{i_3}} \quad \text{(E-Plus)}$$

$$\frac{\boldsymbol{e_1} \Downarrow \boldsymbol{i_1} \qquad \boldsymbol{e_2} \Downarrow \boldsymbol{i_2} \qquad \boldsymbol{i_1}\ \texttt{minus}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}}{\boldsymbol{e_1} - \boldsymbol{e_2} \Downarrow \boldsymbol{i_3}} \quad \text{(E-Minus)}$$

$$\frac{\boldsymbol{e_1} \Downarrow \boldsymbol{i_1} \qquad \boldsymbol{e_2} \Downarrow \boldsymbol{i_2} \qquad \boldsymbol{i_1}\ \texttt{times}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}}{\boldsymbol{e_1} * \boldsymbol{e_2} \Downarrow \boldsymbol{i_3}} \quad \text{(E-Times)}$$

$$\frac{e_1 \Downarrow i_1 \qquad e_2 \Downarrow i_2 \qquad i_1 \texttt{ less than } i_2 \texttt{ is } b_3}{e_1 < e_2 \Downarrow b_3} \quad \text{(E-Lt)}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1 \texttt{ plus } i_2 \texttt{ is } i_3} \quad \text{(B-Plus)}$$

$$\frac{(i_3 = i_1 - i_2)}{i_1 \texttt{ minus } i_2 \texttt{ is } i_3} \quad \text{(B-Minus)}$$

$$\frac{(i_3 = i_1 * i_2)}{i_1 \texttt{ times } i_2 \texttt{ is } i_3} \quad \text{(B-Times)}$$

$$\frac{(b_3 = (i_1 < i_2))}{i_1 \texttt{ less than } i_2 \texttt{ is } b_3} \quad \text{(B-Lt)}$$

$$\frac{e_1 \Downarrow b}{e_1 + e_2 \Downarrow \textbf{error}} \quad \text{(E-PlusBoolL)}$$

$$\frac{e_2 \Downarrow b}{e_1 + e_2 \Downarrow \textbf{error}} \quad \text{(E-PlusBoolR)}$$

$$\frac{e_1 \Downarrow \textbf{error}}{e_1 + e_2 \Downarrow \textbf{error}} \quad \text{(E-PlusErrorL)}$$

$$\frac{e_2 \Downarrow \textbf{error}}{e_1 + e_2 \Downarrow \textbf{error}} \quad \text{(E-PlusErrorR)}$$

$$\frac{e_1 \Downarrow b}{e_1 - e_2 \Downarrow \textbf{error}} \quad \text{(E-MinusBoolL)}$$

$$\frac{e_2 \Downarrow b}{e_1 - e_2 \Downarrow \textbf{error}} \quad \text{(E-MinusBoolR)}$$

$$\frac{e_1 \Downarrow \textbf{error}}{e_1 - e_2 \Downarrow \textbf{error}} \quad \text{(E-MinusErrorL)}$$

$$\frac{e_2 \Downarrow \textbf{error}}{e_1 - e_2 \Downarrow \textbf{error}} \quad \text{(E-MinusErrorR)}$$

$$\frac{e_1 \Downarrow b}{e_1 * e_2 \Downarrow \textbf{error}} \quad \text{(E-TimesBoolL)}$$

$$\frac{e_2 \Downarrow b}{e_1 * e_2 \Downarrow \textbf{error}} \quad \text{(E-TimesBoolR)}$$

$$\frac{e_1 \Downarrow \textbf{error}}{e_1 * e_2 \Downarrow \textbf{error}} \quad \text{(E-TimesErrorL)}$$

$$\frac{e_2 \Downarrow \textbf{error}}{e_1 * e_2 \Downarrow \textbf{error}} \quad \text{(E-TimesErrorR)}$$

$$\frac{e_1 \Downarrow b}{e_1 < e_2 \Downarrow \textbf{error}} \quad \text{(E-LtBoolL)}$$

$$\frac{e_2 \Downarrow b}{e_1 < e_2 \Downarrow \textbf{error}} \quad \text{(E-LtBoolR)}$$

$$\frac{e_1 \Downarrow \textbf{error}}{e_1 < e_2 \Downarrow \textbf{error}} \quad \text{(E-LtErrorL)}$$

$$\frac{e_2 \Downarrow \textbf{error}}{e_1 < e_2 \Downarrow \textbf{error}} \quad \text{(E-LtErrorR)}$$

$$\frac{e_1 \Downarrow i}{\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow \textbf{error}} \quad \text{(E-IfInt)}$$

$$\frac{e_1 \Downarrow \textbf{error}}{\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow \textbf{error}} \quad \text{(E-IfError)}$$

$$\frac{e_1 \Downarrow \texttt{true} \qquad e_2 \Downarrow \textbf{error}}{\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow \textbf{error}} \quad \text{(E-IfTError)}$$

$$\frac{e_1 \Downarrow \texttt{false} \qquad e_3 \Downarrow \textbf{error}}{\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow \textbf{error}} \quad \text{(E-IfFError)}$$

EvalML2

Syntax:

$$
\begin{aligned}
&i \in \textbf{int}\\
&b \in \textbf{bool}\\
&x, y \in \textbf{Var}\\
&v \in \textbf{Value} ::= i \mid b\\
&\mathcal{E} \in \textbf{Env} ::= \bullet \mid \mathcal{E}, x\ \texttt{=}\ v\\
&e \in \textbf{Exp} ::= i \mid b \mid x \mid e\ op\ e \mid \texttt{if}\ e\ \texttt{then}\ e\ \texttt{else}\ e \mid \texttt{let}\ x\ \texttt{=}\ e\ \texttt{in}\ e\\
&op \in \textbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{aligned}
$$

空の環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する.

Judgment Form(s):

$$\mathcal{E} \vdash e \Downarrow v$$

$$i_1\ \texttt{plus}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{minus}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{times}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{less than}\ i_2\ \texttt{is}\ b_3$$

Derivation Rules:

$$\frac{}{\mathcal{E} \vdash i \Downarrow i} \quad \text{(E-Int)}$$

$$\frac{}{\mathcal{E} \vdash b \Downarrow b} \quad \text{(E-Bool)}$$

$$\frac{}{\mathcal{E}, x\ \texttt{=}\ v \vdash x \Downarrow v} \quad \text{(E-Var1)}$$

$$\frac{(y \neq x) \qquad \mathcal{E} \vdash x \Downarrow v_2}{\mathcal{E}, y\ \texttt{=}\ v_1 \vdash x \Downarrow v_2} \quad \text{(E-Var2)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1\ \texttt{plus}\ i_2\ \texttt{is}\ i_3}{\mathcal{E} \vdash e_1\ \texttt{+}\ e_2 \Downarrow i_3} \quad \text{(E-Plus)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1\ \texttt{minus}\ i_2\ \texttt{is}\ i_3}{\mathcal{E} \vdash e_1\ \texttt{-}\ e_2 \Downarrow i_3} \quad \text{(E-Minus)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ times } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ * } e_2 \Downarrow i_3} \tag{E-Times}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ less than } i_2 \texttt{ is } b_3}{\mathcal{E} \vdash e_1 \texttt{ < } e_2 \Downarrow b_3} \tag{E-Lt}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{true} \qquad \mathcal{E} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v} \tag{E-IfT}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{false} \qquad \mathcal{E} \vdash e_3 \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v} \tag{E-IfF}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow v_1 \qquad \mathcal{E}, x \texttt{ = } v_1 \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2 \Downarrow v} \tag{E-Let}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1 \texttt{ plus } i_2 \texttt{ is } i_3} \tag{B-Plus}$$

$$\frac{(i_3 = i_1 - i_2)}{i_1 \texttt{ minus } i_2 \texttt{ is } i_3} \tag{B-Minus}$$

$$\frac{(i_3 = i_1 * i_2)}{i_1 \texttt{ times } i_2 \texttt{ is } i_3} \tag{B-Times}$$

$$\frac{(b_3 = (i_1 < i_2))}{i_1 \texttt{ less than } i_2 \texttt{ is } b_3} \tag{B-Lt}$$

EvalML3

Syntax:

$$
\begin{aligned}
&i \in \textbf{int}\\
&b \in \textbf{bool}\\
&x, y \in \textbf{Var}\\
&v \in \textbf{Value} ::= i \mid b \mid (\mathcal{E})[\texttt{fun } x \to e] \mid (\mathcal{E})[\texttt{rec } x \texttt{ = fun } y \to e]\\
&\mathcal{E} \in \textbf{Env} ::= \bullet \mid \mathcal{E}, x = v\\
&e \in \textbf{Exp} ::= i \mid b \mid x \mid e\ op\ e \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid \texttt{let } x \texttt{ = } e \texttt{ in } e\\
&\qquad\qquad \mid \texttt{fun } x \to e \mid e\ e \mid \texttt{let rec } x \texttt{ = fun } y \to e \texttt{ in } e\\
&op \in \textbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{aligned}
$$

空の環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する.

Judgment Form(s):

$$\mathcal{E} \vdash e \Downarrow v$$

$$i_1 \texttt{ plus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ minus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ times } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ less than } i_2 \texttt{ is } b_3$$

Derivation Rules:

$$\frac{}{\mathcal{E} \vdash i \Downarrow i} \quad \text{(E-Int)}$$

$$\frac{}{\mathcal{E} \vdash b \Downarrow b} \quad \text{(E-Bool)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{true} \qquad \mathcal{E} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v} \quad \text{(E-IfT)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{false} \qquad \mathcal{E} \vdash e_3 \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v} \quad \text{(E-IfF)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ plus } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ + } e_2 \Downarrow i_3} \quad \text{(E-Plus)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ minus } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ - } e_2 \Downarrow i_3} \tag{E-Minus}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ times } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ * } e_2 \Downarrow i_3} \tag{E-Times}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ less than } i_2 \texttt{ is } b_3}{\mathcal{E} \vdash e_1 \texttt{ < } e_2 \Downarrow b_3} \tag{E-Lt}$$

$$\frac{}{\mathcal{E}, x \texttt{ = } v \vdash x \Downarrow v} \tag{E-Var1}$$

$$\frac{(y \neq x) \qquad \mathcal{E} \vdash x \Downarrow v_2}{\mathcal{E}, y \texttt{ = } v_1 \vdash x \Downarrow v_2} \tag{E-Var2}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow v_1 \qquad \mathcal{E}, x \texttt{ = } v_1 \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2 \Downarrow v} \tag{E-Let}$$

$$\frac{}{\mathcal{E} \vdash \texttt{fun } x \texttt{ -> } e \Downarrow (\mathcal{E})\texttt{[fun } x \texttt{ -> } e\texttt{]}} \tag{E-Fun}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)\texttt{[fun } x \texttt{ -> } e_0\texttt{]} \qquad \mathcal{E} \vdash e_2 \Downarrow v_2 \qquad \mathcal{E}_2, x \texttt{ = } v_2 \vdash e_0 \Downarrow v}{\mathcal{E} \vdash e_1\ e_2 \Downarrow v} \tag{E-App}$$

$$\frac{\mathcal{E}, x \texttt{ = } (\mathcal{E})\texttt{[rec } x \texttt{ = fun } y \texttt{ -> } e_1\texttt{]} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{let rec } x \texttt{ = fun } y \texttt{ -> } e_1 \texttt{ in } e_2 \Downarrow v} \tag{E-LetRec}$$

$$\frac{\begin{array}{c}\mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)\texttt{[rec } x \texttt{ = fun } y \texttt{ -> } e_0\texttt{]} \qquad \mathcal{E} \vdash e_2 \Downarrow v_2 \\ \mathcal{E}_2, x \texttt{ = } (\mathcal{E}_2)\texttt{[rec } x \texttt{ = fun } y \texttt{ -> } e_0\texttt{]}, y \texttt{ = } v_2 \vdash e_0 \Downarrow v\end{array}}{\mathcal{E} \vdash e_1\ e_2 \Downarrow v} \tag{E-AppRec}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1 \texttt{ plus } i_2 \texttt{ is } i_3} \tag{B-Plus}$$

$$\frac{(i_3 = i_1 - i_2)}{i_1 \texttt{ minus } i_2 \texttt{ is } i_3} \tag{B-Minus}$$

$$\frac{(i_3 = i_1 * i_2)}{i_1 \texttt{ times } i_2 \texttt{ is } i_3} \tag{B-Times}$$

$$\frac{(b_3 = (i_1 < i_2))}{i_1 \texttt{ less than } i_2 \texttt{ is } b_3} \tag{B-Lt}$$

NamelessML3

Syntax:

$$
\begin{aligned}
&\boldsymbol{i} \in \mathbf{int}\\
&\boldsymbol{b} \in \mathbf{bool}\\
&\boldsymbol{x}, \boldsymbol{y} \in \mathbf{Var}\\
&\boldsymbol{n} \in \mathbf{int}\\
&\boldsymbol{\mathcal{X}} \in \mathbf{VarList} ::= \bullet \mid \boldsymbol{\mathcal{X}}, \boldsymbol{x}\\
&\boldsymbol{e} \in \mathbf{Exp} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid \boldsymbol{x} \mid \boldsymbol{e}\ \boldsymbol{op}\ \boldsymbol{e} \mid \texttt{if}\ \boldsymbol{e}\ \texttt{then}\ \boldsymbol{e}\ \texttt{else}\ \boldsymbol{e} \mid \texttt{let}\ \boldsymbol{x}\ \texttt{=}\ \boldsymbol{e}\ \texttt{in}\ \boldsymbol{e}\\
&\qquad\qquad \mid \texttt{fun}\ \boldsymbol{x} \to \boldsymbol{e} \mid \boldsymbol{e}\ \boldsymbol{e} \mid \texttt{let rec}\ \boldsymbol{x}\ \texttt{=}\ \texttt{fun}\ \boldsymbol{y} \to \boldsymbol{e}\ \texttt{in}\ \boldsymbol{e}\\
&\boldsymbol{d} \in \mathbf{DBExp} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid \texttt{\#}\boldsymbol{n} \mid \boldsymbol{d}\ \boldsymbol{op}\ \boldsymbol{d} \mid \texttt{if}\ \boldsymbol{d}\ \texttt{then}\ \boldsymbol{d}\ \texttt{else}\ \boldsymbol{d} \mid \texttt{let . =}\ \boldsymbol{d}\ \texttt{in}\ \boldsymbol{d}\\
&\qquad\qquad \mid \texttt{fun . →}\ \boldsymbol{d} \mid \boldsymbol{d}\ \boldsymbol{d} \mid \texttt{let rec . = fun . →}\ \boldsymbol{d}\ \texttt{in}\ \boldsymbol{d}\\
&\boldsymbol{op} \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{aligned}
$$

空の環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する.

Judgment Form(s):

$$\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e} \Longrightarrow \boldsymbol{d}$$

Derivation Rules:

$$\frac{}{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{i} \Longrightarrow \boldsymbol{i}} \quad \text{(Tr-Int)}$$

$$\frac{}{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{b} \Longrightarrow \boldsymbol{b}} \quad \text{(Tr-Bool)}$$

$$\frac{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1} \Longrightarrow \boldsymbol{d_1} \qquad \boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_2} \qquad \boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_3} \Longrightarrow \boldsymbol{d_3}}{\boldsymbol{\mathcal{X}} \vdash \texttt{if}\ \boldsymbol{e_1}\ \texttt{then}\ \boldsymbol{e_2}\ \texttt{else}\ \boldsymbol{e_3} \Longrightarrow \texttt{if}\ \boldsymbol{d_1}\ \texttt{then}\ \boldsymbol{d_2}\ \texttt{else}\ \boldsymbol{d_3}} \quad \text{(Tr-If)}$$

$$\frac{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1} \Longrightarrow \boldsymbol{d_1} \qquad \boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_2}}{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1}\ \texttt{+}\ \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_1}\ \texttt{+}\ \boldsymbol{d_2}} \quad \text{(Tr-Plus)}$$

$$\frac{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1} \Longrightarrow \boldsymbol{d_1} \qquad \boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_2}}{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1}\ \texttt{-}\ \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_1}\ \texttt{-}\ \boldsymbol{d_2}} \quad \text{(Tr-Minus)}$$

$$\frac{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1} \Longrightarrow \boldsymbol{d_1} \qquad \boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_2}}{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1}\ \texttt{*}\ \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_1}\ \texttt{*}\ \boldsymbol{d_2}} \quad \text{(Tr-Times)}$$

$$\frac{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1} \Longrightarrow \boldsymbol{d_1} \qquad \boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_2}}{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1}\ \texttt{<}\ \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_1}\ \texttt{<}\ \boldsymbol{d_2}} \quad \text{(Tr-Lt)}$$

$$\frac{(\boldsymbol{n} = 1)}{\boldsymbol{\mathcal{X}}, \boldsymbol{x} \vdash \boldsymbol{x} \Longrightarrow \texttt{\#}\boldsymbol{n}} \quad \text{(Tr-Var1)}$$

$$\frac{(\boldsymbol{y} \neq \boldsymbol{x}) \qquad \boldsymbol{\mathcal{X}} \vdash \boldsymbol{x} \Longrightarrow \texttt{\#}\boldsymbol{n_1} \qquad (\boldsymbol{n_2} = \boldsymbol{n_1} + 1)}{\boldsymbol{\mathcal{X}}, \boldsymbol{y} \vdash \boldsymbol{x} \Longrightarrow \texttt{\#}\boldsymbol{n_2}} \quad \text{(Tr-Var2)}$$

$$\frac{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1} \Longrightarrow \boldsymbol{d_1} \qquad \boldsymbol{\mathcal{X}}, \boldsymbol{x} \vdash \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_2}}{\boldsymbol{\mathcal{X}} \vdash \texttt{let}\ \boldsymbol{x}\ \texttt{=}\ \boldsymbol{e_1}\ \texttt{in}\ \boldsymbol{e_2} \Longrightarrow \texttt{let .}\ \texttt{=}\ \boldsymbol{d_1}\ \texttt{in}\ \boldsymbol{d_2}} \quad \text{(Tr-Let)}$$

$$\frac{\boldsymbol{\mathcal{X}}, \boldsymbol{x} \vdash \boldsymbol{e} \Longrightarrow \boldsymbol{d}}{\boldsymbol{\mathcal{X}} \vdash \texttt{fun}\ \boldsymbol{x} \rightarrow \boldsymbol{e} \Longrightarrow \texttt{fun .} \rightarrow \boldsymbol{d}} \quad \text{(Tr-Fun)}$$

$$\frac{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1} \Longrightarrow \boldsymbol{d_1} \qquad \boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_2}}{\boldsymbol{\mathcal{X}} \vdash \boldsymbol{e_1}\ \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_1}\ \boldsymbol{d_2}} \quad \text{(Tr-App)}$$

$$\frac{\boldsymbol{\mathcal{X}}, \boldsymbol{x}, \boldsymbol{y} \vdash \boldsymbol{e_1} \Longrightarrow \boldsymbol{d_1} \qquad \boldsymbol{\mathcal{X}}, \boldsymbol{x} \vdash \boldsymbol{e_2} \Longrightarrow \boldsymbol{d_2}}{\boldsymbol{\mathcal{X}} \vdash \texttt{let rec}\ \boldsymbol{x}\ \texttt{=}\ \texttt{fun}\ \boldsymbol{y} \rightarrow \boldsymbol{e_1}\ \texttt{in}\ \boldsymbol{e_2} \Longrightarrow \texttt{let rec .}\ \texttt{=}\ \texttt{fun .} \rightarrow \boldsymbol{d_1}\ \texttt{in}\ \boldsymbol{d_2}} \quad \text{(Tr-LetRec)}$$

EvalNamelessML3

Syntax:

$$
\begin{aligned}
&\boldsymbol{i} \in \mathbf{int}\\
&\boldsymbol{b} \in \mathbf{bool}\\
&\boldsymbol{n} \in \mathbf{int}\\
&\boldsymbol{w} \in \mathbf{DBValue} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid (\boldsymbol{\mathcal{V}})\texttt{[fun . } \rightarrow \boldsymbol{d}\texttt{]} \mid (\boldsymbol{\mathcal{V}})\texttt{[rec . = fun . } \rightarrow \boldsymbol{d}\texttt{]}\\
&\boldsymbol{\mathcal{V}} \in \mathbf{DBValueList} ::= \bullet \mid \boldsymbol{\mathcal{V}}, \boldsymbol{w}\\
&\boldsymbol{d} \in \mathbf{DBExp} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid \texttt{\#}\boldsymbol{n} \mid \boldsymbol{d}\ \boldsymbol{op}\ \boldsymbol{d} \mid \texttt{if } \boldsymbol{d} \texttt{ then } \boldsymbol{d} \texttt{ else } \boldsymbol{d} \mid \texttt{let . = } \boldsymbol{d} \texttt{ in } \boldsymbol{d}\\
&\qquad\qquad \mid \texttt{fun . } \rightarrow \boldsymbol{d} \mid \boldsymbol{d}\ \boldsymbol{d} \mid \texttt{let rec . = fun . } \rightarrow \boldsymbol{d} \texttt{ in } \boldsymbol{d}\\
&\boldsymbol{op} \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{aligned}
$$

空の環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する.

Judgment Form(s):

$$\boldsymbol{\mathcal{V}} \vdash \boldsymbol{d} \Downarrow \boldsymbol{w}$$

$$\boldsymbol{i_1} \texttt{ plus } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}$$

$$\boldsymbol{i_1} \texttt{ minus } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}$$

$$\boldsymbol{i_1} \texttt{ times } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}$$

$$\boldsymbol{i_1} \texttt{ less than } \boldsymbol{i_2} \texttt{ is } \boldsymbol{b_3}$$

Derivation Rules:

$$\frac{}{\boldsymbol{\mathcal{V}} \vdash \boldsymbol{i} \Downarrow \boldsymbol{i}} \quad \text{(E-Int)}$$

$$\frac{}{\boldsymbol{\mathcal{V}} \vdash \boldsymbol{b} \Downarrow \boldsymbol{b}} \quad \text{(E-Bool)}$$

$$\frac{\boldsymbol{\mathcal{V}} \vdash \boldsymbol{d_1} \Downarrow \texttt{true} \qquad \boldsymbol{\mathcal{V}} \vdash \boldsymbol{d_2} \Downarrow \boldsymbol{w}}{\boldsymbol{\mathcal{V}} \vdash \texttt{if } \boldsymbol{d_1} \texttt{ then } \boldsymbol{d_2} \texttt{ else } \boldsymbol{d_3} \Downarrow \boldsymbol{w}} \quad \text{(E-IfT)}$$

$$\frac{\boldsymbol{\mathcal{V}} \vdash \boldsymbol{d_1} \Downarrow \texttt{false} \qquad \boldsymbol{\mathcal{V}} \vdash \boldsymbol{d_3} \Downarrow \boldsymbol{w}}{\boldsymbol{\mathcal{V}} \vdash \texttt{if } \boldsymbol{d_1} \texttt{ then } \boldsymbol{d_2} \texttt{ else } \boldsymbol{d_3} \Downarrow \boldsymbol{w}} \quad \text{(E-IfF)}$$

$$\frac{\boldsymbol{\mathcal{V}} \vdash \boldsymbol{d_1} \Downarrow \boldsymbol{i_1} \qquad \boldsymbol{\mathcal{V}} \vdash \boldsymbol{d_2} \Downarrow \boldsymbol{i_2} \qquad \boldsymbol{i_1} \texttt{ plus } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}}{\boldsymbol{\mathcal{V}} \vdash \boldsymbol{d_1} \texttt{ + } \boldsymbol{d_2} \Downarrow \boldsymbol{i_3}} \quad \text{(E-Plus)}$$

$$\frac{\mathcal{V} \vdash d_1 \Downarrow i_1 \qquad \mathcal{V} \vdash d_2 \Downarrow i_2 \qquad i_1 \texttt{ minus } i_2 \texttt{ is } i_3}{\mathcal{V} \vdash d_1 \texttt{ - } d_2 \Downarrow i_3} \tag{E-Minus}$$

$$\frac{\mathcal{V} \vdash d_1 \Downarrow i_1 \qquad \mathcal{V} \vdash d_2 \Downarrow i_2 \qquad i_1 \texttt{ times } i_2 \texttt{ is } i_3}{\mathcal{V} \vdash d_1 \texttt{ * } d_2 \Downarrow i_3} \tag{E-Times}$$

$$\frac{\mathcal{V} \vdash d_1 \Downarrow i_1 \qquad \mathcal{V} \vdash d_2 \Downarrow i_2 \qquad i_1 \texttt{ less than } i_2 \texttt{ is } b_3}{\mathcal{V} \vdash d_1 \texttt{ < } d_2 \Downarrow b_3} \tag{E-Lt}$$

$$\frac{(\mathcal{V}[n] = w)}{\mathcal{V} \vdash \texttt{\#}n \Downarrow w} \tag{E-Var}$$

$$\frac{\mathcal{V} \vdash d_1 \Downarrow w_1 \qquad \mathcal{V}, w_1 \vdash d_2 \Downarrow w}{\mathcal{V} \vdash \texttt{let . = } d_1 \texttt{ in } d_2 \Downarrow w} \tag{E-Let}$$

$$\frac{}{\mathcal{V} \vdash \texttt{fun . } \to d \Downarrow (\mathcal{V})\texttt{[fun . } \to d\texttt{]}} \tag{E-Fun}$$

$$\frac{\mathcal{V} \vdash d_1 \Downarrow (\mathcal{V}_2)\texttt{[fun . } \to d_0\texttt{]} \qquad \mathcal{V} \vdash d_2 \Downarrow w_2 \qquad \mathcal{V}_2, w_2 \vdash d_0 \Downarrow w}{\mathcal{V} \vdash d_1\ d_2 \Downarrow w} \tag{E-App}$$

$$\frac{\mathcal{V}, (\mathcal{V})\texttt{[rec . = fun . } \to d_1\texttt{]} \vdash d_2 \Downarrow w}{\mathcal{V} \vdash \texttt{let rec . = fun . } \to d_1 \texttt{ in } d_2 \Downarrow w} \tag{E-LetRec}$$

$$\frac{\begin{array}{c}\mathcal{V} \vdash d_1 \Downarrow (\mathcal{V}_2)\texttt{[rec . = fun . } \to d_0\texttt{]} \qquad \mathcal{V} \vdash d_2 \Downarrow w_2 \\ \mathcal{V}_2, (\mathcal{V}_2)\texttt{[rec . = fun . } \to d_0\texttt{]}, w_2 \vdash d_0 \Downarrow w\end{array}}{\mathcal{V} \vdash d_1\ d_2 \Downarrow w} \tag{E-AppRec}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1 \texttt{ plus } i_2 \texttt{ is } i_3} \tag{B-Plus}$$

$$\frac{(i_3 = i_1 - i_2)}{i_1 \texttt{ minus } i_2 \texttt{ is } i_3} \tag{B-Minus}$$

$$\frac{(i_3 = i_1 * i_2)}{i_1 \texttt{ times } i_2 \texttt{ is } i_3} \tag{B-Times}$$

$$\frac{(b_3 = (i_1 < i_2))}{i_1 \texttt{ less than } i_2 \texttt{ is } b_3} \tag{B-Lt}$$

EvalML4

Syntax:

$$
\begin{aligned}
&i \in \mathbf{int}\\
&b \in \mathbf{bool}\\
&x, y \in \mathbf{Var}\\
&v \in \mathbf{Value} ::= i \mid b \mid (\mathcal{E})\texttt{[fun } x \to e\texttt{]} \mid (\mathcal{E})\texttt{[rec } x \texttt{ = fun } y \to e\texttt{]} \mid \texttt{[]} \mid v \texttt{ :: } v\\
&\mathcal{E} \in \mathbf{Env} ::= \bullet \mid \mathcal{E}, x \texttt{ = } v\\
&e \in \mathbf{Exp} ::= i \mid b \mid x \mid e\ op\ e \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid \texttt{let } x \texttt{ = } e \texttt{ in } e\\
&\qquad\qquad \mid \texttt{fun } x \to e \mid e\ e \mid \texttt{let rec } x \texttt{ = fun } y \to e \texttt{ in } e\\
&\qquad\qquad \mid \texttt{[]} \mid e \texttt{ :: } e \mid \texttt{match } e \texttt{ with [] } \to e \texttt{ | } x \texttt{ :: } y \to e\\
&op \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{aligned}
$$

空の環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する.

Judgment Form(s):

$$\mathcal{E} \vdash e \Downarrow v$$

$$i_1 \texttt{ plus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ minus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ times } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ less than } i_2 \texttt{ is } b_3$$

Derivation Rules:

$$\frac{}{\mathcal{E} \vdash i \Downarrow i} \quad \text{(E-Int)}$$

$$\frac{}{\mathcal{E} \vdash b \Downarrow b} \quad \text{(E-Bool)}$$

$$\frac{(\mathcal{E}(x) = v)}{\mathcal{E} \vdash x \Downarrow v} \quad \text{(E-Var)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ plus } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ + } e_2 \Downarrow i_3} \quad \text{(E-Plus)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ minus } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ - } e_2 \Downarrow i_3} \quad \text{(E-Minus)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ times } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 * e_2 \Downarrow i_3} \tag{E-Times}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ less than } i_2 \texttt{ is } b_3}{\mathcal{E} \vdash e_1 < e_2 \Downarrow b_3} \tag{E-Lt}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{true} \qquad \mathcal{E} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v} \tag{E-IfT}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{false} \qquad \mathcal{E} \vdash e_3 \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v} \tag{E-IfF}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow v_1 \qquad \mathcal{E}, x = v_1 \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2 \Downarrow v} \tag{E-Let}$$

$$\frac{}{\mathcal{E} \vdash \texttt{fun } x \to e \Downarrow (\mathcal{E})\texttt{[fun } x \to e\texttt{]}} \tag{E-Fun}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)\texttt{[fun } x \to e_0\texttt{]} \qquad \mathcal{E} \vdash e_2 \Downarrow v_2 \qquad \mathcal{E}_2, x = v_2 \vdash e_0 \Downarrow v}{\mathcal{E} \vdash e_1\ e_2 \Downarrow v} \tag{E-App}$$

$$\frac{\mathcal{E}, x = (\mathcal{E})\texttt{[rec } x \texttt{ = fun } y \to e_1\texttt{]} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{let rec } x \texttt{ = fun } y \to e_1 \texttt{ in } e_2 \Downarrow v} \tag{E-LetRec}$$

$$\frac{\begin{array}{c}\mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)\texttt{[rec } x \texttt{ = fun } y \to e_0\texttt{]} \qquad \mathcal{E} \vdash e_2 \Downarrow v_2 \\ \mathcal{E}_2, x = (\mathcal{E}_2)\texttt{[rec } x \texttt{ = fun } y \to e_0\texttt{]}, y = v_2 \vdash e_0 \Downarrow v\end{array}}{\mathcal{E} \vdash e_1\ e_2 \Downarrow v} \tag{E-AppRec}$$

$$\frac{}{\mathcal{E} \vdash \texttt{[]} \Downarrow \texttt{[]}} \tag{E-Nil}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow v_1 \qquad \mathcal{E} \vdash e_2 \Downarrow v_2}{\mathcal{E} \vdash e_1 :: e_2 \Downarrow v_1 :: v_2} \tag{E-Cons}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{[]} \qquad \mathcal{E} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{match } e_1 \texttt{ with [] } \to e_2 \texttt{ | } x :: y \to e_3 \Downarrow v} \tag{E-MatchNil}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow v_1 :: v_2 \qquad \mathcal{E}, x = v_1, y = v_2 \vdash e_3 \Downarrow v}{\mathcal{E} \vdash \texttt{match } e_1 \texttt{ with [] } \to e_2 \texttt{ | } x :: y \to e_3 \Downarrow v} \tag{E-MatchCons}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1 \texttt{ plus } i_2 \texttt{ is } i_3} \tag{B-Plus}$$

$$\frac{(\boldsymbol{i_3} = \boldsymbol{i_1} - \boldsymbol{i_2})}{\boldsymbol{i_1}\ \texttt{minus}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}} \quad \text{(B-Minus)}$$

$$\frac{(\boldsymbol{i_3} = \boldsymbol{i_1} * \boldsymbol{i_2})}{\boldsymbol{i_1}\ \texttt{times}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}} \quad \text{(B-Times)}$$

$$\frac{(\boldsymbol{b_3} = (\boldsymbol{i_1} < \boldsymbol{i_2}))}{\boldsymbol{i_1}\ \texttt{less than}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{b_3}} \quad \text{(B-Lt)}$$

EvalML5

Syntax:

$$
\begin{aligned}
&\boldsymbol{i} \in \mathbf{int}\\
&\boldsymbol{b} \in \mathbf{bool}\\
&\boldsymbol{x}, \boldsymbol{y} \in \mathbf{Var}\\
&\boldsymbol{v} \in \mathbf{Value} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid \texttt{(}\boldsymbol{\mathcal{E}}\texttt{)[fun } \boldsymbol{x} \texttt{ -> } \boldsymbol{e}\texttt{]} \mid \texttt{(}\boldsymbol{\mathcal{E}}\texttt{)[rec } \boldsymbol{x} \texttt{ = fun } \boldsymbol{y} \texttt{ -> } \boldsymbol{e}\texttt{]} \mid \texttt{[]} \mid \boldsymbol{v} \texttt{ :: } \boldsymbol{v}\\
&\boldsymbol{\mathcal{E}} \in \mathbf{Env} ::= \bullet \mid \boldsymbol{\mathcal{E}}, \boldsymbol{x} \texttt{ = } \boldsymbol{v}\\
&\boldsymbol{p} \in \mathbf{Pat} ::= \boldsymbol{x} \mid \texttt{[]} \mid \boldsymbol{p} \texttt{ :: } \boldsymbol{p} \mid \texttt{\_}\\
&\boldsymbol{res} \in \mathbf{Res} ::= \boldsymbol{\mathcal{E}} \mid F\\
&\boldsymbol{c} \in \mathbf{Clauses} ::= \boldsymbol{p} \texttt{ -> } \boldsymbol{e} \mid \boldsymbol{p} \texttt{ -> } \boldsymbol{e} \texttt{ | } \boldsymbol{c}\\
&\boldsymbol{e} \in \mathbf{Exp} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid \boldsymbol{x} \mid \boldsymbol{e}\ \boldsymbol{op}\ \boldsymbol{e} \mid \texttt{if } \boldsymbol{e} \texttt{ then } \boldsymbol{e} \texttt{ else } \boldsymbol{e} \mid \texttt{let } \boldsymbol{x} \texttt{ = } \boldsymbol{e} \texttt{ in } \boldsymbol{e}\\
&\qquad\qquad \mid \texttt{fun } \boldsymbol{x} \texttt{ -> } \boldsymbol{e} \mid \boldsymbol{e}\ \boldsymbol{e} \mid \texttt{let rec } \boldsymbol{x} \texttt{ = fun } \boldsymbol{y} \texttt{ -> } \boldsymbol{e} \texttt{ in } \boldsymbol{e}\\
&\qquad\qquad \mid \texttt{[]} \mid \boldsymbol{e} \texttt{ :: } \boldsymbol{e} \mid \texttt{match } \boldsymbol{e} \texttt{ with } \boldsymbol{c}\\
&\boldsymbol{op} \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{aligned}
$$

空の環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する.

Judgment Form(s):

$$\boldsymbol{p} \texttt{ matches } \boldsymbol{v} \texttt{ when } (\boldsymbol{\mathcal{E}})$$

$$\boldsymbol{p} \texttt{ doesn't match } \boldsymbol{v}$$

$$\boldsymbol{\mathcal{E}} \vdash \boldsymbol{e} \Downarrow \boldsymbol{v}$$

$$\boldsymbol{i_1} \texttt{ plus } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}$$

$$\boldsymbol{i_1} \texttt{ minus } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}$$

$$\boldsymbol{i_1} \texttt{ times } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}$$

$$\boldsymbol{i_1} \texttt{ less than } \boldsymbol{i_2} \texttt{ is } \boldsymbol{b_3}$$

Derivation Rules:

$$\frac{}{\boldsymbol{x} \texttt{ matches } \boldsymbol{v} \texttt{ when } (\boldsymbol{x} \texttt{ = } \boldsymbol{v})} \quad \text{(M-Var)}$$

$$\frac{}{\texttt{[] matches [] when ()}} \quad \text{(M-Nil)}$$

$$\frac{\boldsymbol{p_1} \texttt{ matches } \boldsymbol{v_1} \texttt{ when } (\boldsymbol{\mathcal{E}_1}) \qquad \boldsymbol{p_2} \texttt{ matches } \boldsymbol{v_2} \texttt{ when } (\boldsymbol{\mathcal{E}_2}) \qquad (\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}_1} \uplus \boldsymbol{\mathcal{E}_2})}{\boldsymbol{p_1} \texttt{ :: } \boldsymbol{p_2} \texttt{ matches } \boldsymbol{v_1} \texttt{ :: } \boldsymbol{v_2} \texttt{ when } (\boldsymbol{\mathcal{E}})} \quad \text{(M-Cons)}$$

$$\frac{}{\texttt{\_ matches } v \texttt{ when } ()} \quad \text{(M-Wild)}$$

$$\frac{}{\texttt{[] doesn't match } v_1 \texttt{ :: } v_2} \quad \text{(NM-ConsNil)}$$

$$\frac{}{p_1 \texttt{ :: } p_2 \texttt{ doesn't match []}} \quad \text{(NM-NilCons)}$$

$$\frac{p_1 \texttt{ doesn't match } v_1}{p_1 \texttt{ :: } p_2 \texttt{ doesn't match } v_1 \texttt{ :: } v_2} \quad \text{(NM-ConsConsL)}$$

$$\frac{p_2 \texttt{ doesn't match } v_2}{p_1 \texttt{ :: } p_2 \texttt{ doesn't match } v_1 \texttt{ :: } v_2} \quad \text{(NM-ConsConsR)}$$

$$\frac{}{\mathcal{E} \vdash i \Downarrow i} \quad \text{(E-Int)}$$

$$\frac{}{\mathcal{E} \vdash b \Downarrow b} \quad \text{(E-Bool)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{true} \qquad \mathcal{E} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v} \quad \text{(E-IfT)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow \texttt{false} \qquad \mathcal{E} \vdash e_3 \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v} \quad \text{(E-IfF)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ plus } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ + } e_2 \Downarrow i_3} \quad \text{(E-Plus)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ minus } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ - } e_2 \Downarrow i_3} \quad \text{(E-Minus)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ times } i_2 \texttt{ is } i_3}{\mathcal{E} \vdash e_1 \texttt{ * } e_2 \Downarrow i_3} \quad \text{(E-Times)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \qquad \mathcal{E} \vdash e_2 \Downarrow i_2 \qquad i_1 \texttt{ less than } i_2 \texttt{ is } b_3}{\mathcal{E} \vdash e_1 \texttt{ < } e_2 \Downarrow b_3} \quad \text{(E-Lt)}$$

$$\frac{(\mathcal{E}(x) = v)}{\mathcal{E} \vdash x \Downarrow v} \quad \text{(E-Var)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow v_1 \qquad \mathcal{E}, x \texttt{ = } v_1 \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2 \Downarrow v} \quad \text{(E-Let)}$$

$$\frac{}{\mathcal{E} \vdash \texttt{fun}\ x \to e \Downarrow (\mathcal{E})[\texttt{fun}\ x \to e]} \quad \text{(E-Fun)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)[\texttt{fun}\ x \to e_0] \qquad \mathcal{E} \vdash e_2 \Downarrow v_2 \qquad \mathcal{E}_2, x = v_2 \vdash e_0 \Downarrow v}{\mathcal{E} \vdash e_1\ e_2 \Downarrow v} \quad \text{(E-App)}$$

$$\frac{\mathcal{E}, x = (\mathcal{E})[\texttt{rec}\ x\ =\ \texttt{fun}\ y \to e_1] \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \texttt{let rec}\ x\ =\ \texttt{fun}\ y \to e_1\ \texttt{in}\ e_2 \Downarrow v} \quad \text{(E-LetRec)}$$

$$\frac{\begin{array}{c}\mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)[\texttt{rec}\ x\ =\ \texttt{fun}\ y \to e_0] \qquad \mathcal{E} \vdash e_2 \Downarrow v_2 \\ \mathcal{E}_2, x = (\mathcal{E}_2)[\texttt{rec}\ x\ =\ \texttt{fun}\ y \to e_0], y = v_2 \vdash e_0 \Downarrow v\end{array}}{\mathcal{E} \vdash e_1\ e_2 \Downarrow v} \quad \text{(E-AppRec)}$$

$$\frac{}{\mathcal{E} \vdash [] \Downarrow []} \quad \text{(E-Nil)}$$

$$\frac{\mathcal{E} \vdash e_1 \Downarrow v_1 \qquad \mathcal{E} \vdash e_2 \Downarrow v_2}{\mathcal{E} \vdash e_1 :: e_2 \Downarrow v_1 :: v_2} \quad \text{(E-Cons)}$$

$$\frac{\mathcal{E} \vdash e_0 \Downarrow v \qquad p\ \texttt{matches}\ v\ \texttt{when}\ (\mathcal{E}_1) \qquad (\mathcal{E}_2 = \mathcal{E}; \mathcal{E}_1) \qquad \mathcal{E}_2 \vdash e \Downarrow v'}{\mathcal{E} \vdash \texttt{match}\ e_0\ \texttt{with}\ p \to e \Downarrow v'} \quad \text{(E-MatchM1)}$$

$$\frac{\mathcal{E} \vdash e_0 \Downarrow v \qquad p\ \texttt{matches}\ v\ \texttt{when}\ (\mathcal{E}_1) \qquad (\mathcal{E}_2 = \mathcal{E}; \mathcal{E}_1) \qquad \mathcal{E}_2 \vdash e \Downarrow v'}{\mathcal{E} \vdash \texttt{match}\ e_0\ \texttt{with}\ p \to e\ \texttt{|}\ c \Downarrow v'} \quad \text{(E-MatchM2)}$$

$$\frac{\mathcal{E} \vdash e_0 \Downarrow v \qquad p\ \texttt{doesn't match}\ v \qquad \mathcal{E} \vdash \texttt{match}\ e_0\ \texttt{with}\ c \Downarrow v'}{\mathcal{E} \vdash \texttt{match}\ e_0\ \texttt{with}\ p \to e\ \texttt{|}\ c \Downarrow v'} \quad \text{(E-MatchN)}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1\ \texttt{plus}\ i_2\ \texttt{is}\ i_3} \quad \text{(B-Plus)}$$

$$\frac{(i_3 = i_1 - i_2)}{i_1\ \texttt{minus}\ i_2\ \texttt{is}\ i_3} \quad \text{(B-Minus)}$$

$$\frac{(i_3 = i_1 * i_2)}{i_1\ \texttt{times}\ i_2\ \texttt{is}\ i_3} \quad \text{(B-Times)}$$

$$\frac{(b_3 = (i_1 < i_2))}{i_1\ \texttt{less than}\ i_2\ \texttt{is}\ b_3} \quad \text{(B-Lt)}$$

TypingML4

Syntax:

$$
\begin{aligned}
&\boldsymbol{i} \in \mathbf{int}\\
&\boldsymbol{b} \in \mathbf{bool}\\
&\boldsymbol{x}, \boldsymbol{y} \in \mathbf{Var}\\
&\boldsymbol{\tau} \in \mathbf{Types} ::= \texttt{bool} \mid \texttt{int} \mid \boldsymbol{\tau} \to \boldsymbol{\tau} \mid \boldsymbol{\tau}\ \texttt{list}\\
&\boldsymbol{\Gamma} \in \mathbf{Env} ::= \bullet \mid \boldsymbol{\Gamma}, \boldsymbol{x} : \boldsymbol{\tau}\\
&\boldsymbol{e} \in \mathbf{Exp} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid \boldsymbol{x} \mid \boldsymbol{e}\ \boldsymbol{op}\ \boldsymbol{e} \mid \texttt{if}\ \boldsymbol{e}\ \texttt{then}\ \boldsymbol{e}\ \texttt{else}\ \boldsymbol{e} \mid \texttt{let}\ \boldsymbol{x}\ \texttt{=}\ \boldsymbol{e}\ \texttt{in}\ \boldsymbol{e}\\
&\qquad\qquad \mid \texttt{fun}\ \boldsymbol{x} \to \boldsymbol{e} \mid \boldsymbol{e}\ \boldsymbol{e} \mid \texttt{let rec}\ \boldsymbol{x}\ \texttt{=}\ \texttt{fun}\ \boldsymbol{y} \to \boldsymbol{e}\ \texttt{in}\ \boldsymbol{e}\\
&\qquad\qquad \mid \texttt{[]} \mid \boldsymbol{e}\ \texttt{::}\ \boldsymbol{e} \mid \texttt{match}\ \boldsymbol{e}\ \texttt{with []} \to \boldsymbol{e}\ \texttt{|}\ \boldsymbol{x}\ \texttt{::}\ \boldsymbol{y} \to \boldsymbol{e}\\
&\boldsymbol{op} \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{aligned}
$$

空の型環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する.

Judgment Form(s):

$$\boldsymbol{\Gamma} \vdash \boldsymbol{e} : \boldsymbol{\tau}$$

Derivation Rules:

$$\frac{}{\boldsymbol{\Gamma} \vdash \boldsymbol{i} : \texttt{int}} \quad \text{(T-Int)}$$

$$\frac{}{\boldsymbol{\Gamma} \vdash \boldsymbol{b} : \texttt{bool}} \quad \text{(T-Bool)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{bool} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \boldsymbol{\tau} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_3} : \boldsymbol{\tau}}{\boldsymbol{\Gamma} \vdash \texttt{if}\ \boldsymbol{e_1}\ \texttt{then}\ \boldsymbol{e_2}\ \texttt{else}\ \boldsymbol{e_3} : \boldsymbol{\tau}} \quad \text{(T-If)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{int} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \texttt{int}}{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1}\ \texttt{+}\ \boldsymbol{e_2} : \texttt{int}} \quad \text{(T-Plus)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{int} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \texttt{int}}{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1}\ \texttt{-}\ \boldsymbol{e_2} : \texttt{int}} \quad \text{(T-Minus)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{int} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \texttt{int}}{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1}\ \texttt{*}\ \boldsymbol{e_2} : \texttt{int}} \quad \text{(T-Times)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{int} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \texttt{int}}{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1}\ \texttt{<}\ \boldsymbol{e_2} : \texttt{bool}} \quad \text{(T-Lt)}$$

$$\frac{(\Gamma(x) = \tau)}{\Gamma \vdash x : \tau} \quad \text{(T-Var)}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2 : \tau_2} \quad \text{(T-Let)}$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \texttt{fun } x \to e : \tau_1 \to \tau_2} \quad \text{(T-Fun)}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\ e_2 : \tau_2} \quad \text{(T-App)}$$

$$\frac{\Gamma, x : \tau_1 \to \tau_2, y : \tau_1 \vdash e_1 : \tau_2 \qquad \Gamma, x : \tau_1 \to \tau_2 \vdash e_2 : \tau}{\Gamma \vdash \texttt{let rec } x \texttt{ = fun } y \to e_1 \texttt{ in } e_2 : \tau} \quad \text{(T-LetRec)}$$

$$\frac{}{\Gamma \vdash \texttt{[]} : \tau\ \texttt{list}} \quad \text{(T-Nil)}$$

$$\frac{\Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau\ \texttt{list}}{\Gamma \vdash e_1 :: e_2 : \tau\ \texttt{list}} \quad \text{(T-Cons)}$$

$$\frac{\Gamma \vdash e_1 : \tau'\ \texttt{list} \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma, x : \tau', y : \tau'\ \texttt{list} \vdash e_3 : \tau}{\Gamma \vdash \texttt{match } e_1 \texttt{ with [] } \to e_2 \texttt{ | } x :: y \to e_3 : \tau} \quad \text{(T-Match)}$$

PolyTypingML4

Syntax:

$$
\begin{aligned}
&\boldsymbol{i} \in \mathbf{int}\\
&\boldsymbol{b} \in \mathbf{bool}\\
&\boldsymbol{x}, \boldsymbol{y} \in \mathbf{Var}\\
&\boldsymbol{\alpha} \in \mathbf{TVar}\\
&\boldsymbol{\tau} \in \mathbf{Types} ::= \boldsymbol{\alpha} \mid \boldsymbol{i} \mid \texttt{bool} \mid \texttt{int} \mid \boldsymbol{\tau} \to \boldsymbol{\tau} \mid \boldsymbol{\tau}\ \texttt{list}\\
&\boldsymbol{\sigma} \in \mathbf{TyScheme} ::= \boldsymbol{\tau} \mid \boldsymbol{i}.\boldsymbol{\tau}\\
&\boldsymbol{\Gamma} \in \mathbf{Env} ::= \bullet \mid \boldsymbol{\Gamma}, \boldsymbol{x} : \boldsymbol{\sigma}\\
&\boldsymbol{e} \in \mathbf{Exp} ::= \boldsymbol{i} \mid \boldsymbol{b} \mid \boldsymbol{x} \mid \boldsymbol{e}\ \boldsymbol{op}\ \boldsymbol{e} \mid \texttt{if}\ \boldsymbol{e}\ \texttt{then}\ \boldsymbol{e}\ \texttt{else}\ \boldsymbol{e} \mid \texttt{let}\ \boldsymbol{x}\ \texttt{=}\ \boldsymbol{e}\ \texttt{in}\ \boldsymbol{e}\\
&\qquad\qquad \mid \texttt{fun}\ \boldsymbol{x} \to \boldsymbol{e} \mid \boldsymbol{e}\ \boldsymbol{e} \mid \texttt{let rec}\ \boldsymbol{x}\ \texttt{=}\ \texttt{fun}\ \boldsymbol{y} \to \boldsymbol{e}\ \texttt{in}\ \boldsymbol{e}\\
&\qquad\qquad \mid \texttt{[]} \mid \boldsymbol{e}\ \texttt{::}\ \boldsymbol{e} \mid \texttt{match}\ \boldsymbol{e}\ \texttt{with []} \to \boldsymbol{e}\ \texttt{|}\ \boldsymbol{x}\ \texttt{::}\ \boldsymbol{y} \to \boldsymbol{e}\\
&\boldsymbol{op} \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{aligned}
$$

空の型環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する.

Judgment Form(s):

$$\boldsymbol{\Gamma} \vdash \boldsymbol{e} : \boldsymbol{\tau}$$

Derivation Rules:

$$\frac{}{\boldsymbol{\Gamma} \vdash \boldsymbol{i} : \texttt{int}} \quad \text{(T-Int)}$$

$$\frac{}{\boldsymbol{\Gamma} \vdash \boldsymbol{b} : \texttt{bool}} \quad \text{(T-Bool)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{bool} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \boldsymbol{\tau} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_3} : \boldsymbol{\tau}}{\boldsymbol{\Gamma} \vdash \texttt{if}\ \boldsymbol{e_1}\ \texttt{then}\ \boldsymbol{e_2}\ \texttt{else}\ \boldsymbol{e_3} : \boldsymbol{\tau}} \quad \text{(T-If)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{int} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \texttt{int}}{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1}\ \texttt{+}\ \boldsymbol{e_2} : \texttt{int}} \quad \text{(T-Plus)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{int} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \texttt{int}}{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1}\ \texttt{-}\ \boldsymbol{e_2} : \texttt{int}} \quad \text{(T-Minus)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{int} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \texttt{int}}{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1}\ \texttt{*}\ \boldsymbol{e_2} : \texttt{int}} \quad \text{(T-Times)}$$

$$\frac{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1} : \texttt{int} \qquad \boldsymbol{\Gamma} \vdash \boldsymbol{e_2} : \texttt{int}}{\boldsymbol{\Gamma} \vdash \boldsymbol{e_1}\ \texttt{<}\ \boldsymbol{e_2} : \texttt{bool}} \quad \text{(T-Lt)}$$

$$\frac{(\Gamma(x) = \sigma) \qquad (\sigma \succeq \tau)}{\Gamma \vdash x : \tau} \tag{T-Var}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x\colon \sigma \vdash e_2 : \tau_2 \qquad (\sigma = \alpha_1 \cdots \alpha_n.\tau_1 \text{ and } \{\alpha_1, \ldots, \alpha_n \cap FTV(\Gamma) = \emptyset)}{\Gamma \vdash \texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2 : \tau_2} \tag{T-Let}$$

$$\frac{\Gamma, x\colon \tau_1 \vdash e : \tau_2}{\Gamma \vdash \texttt{fun } x \to e : \tau_1 \to \tau_2} \tag{T-Abs}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\ e_2 : \tau_2} \tag{T-App}$$

$$\frac{\Gamma, x\colon \tau_1 \to \tau_2, y\colon \tau_1 \vdash e_1 : \tau_2 \qquad \Gamma, x\colon \sigma \vdash e_2 : \tau \qquad (\sigma = \alpha_1 \cdots \alpha_n.\tau_1 \to \tau_2 \text{ and } \{\alpha_1, \ldots, \alpha_n\} \cap FTV(\Gamma) = \emptyset)}{\Gamma \vdash \texttt{let rec } x \texttt{ = fun } y \to e_1 \texttt{ in } e_2 : \tau} \tag{T-LetRec}$$

$$\frac{}{\Gamma \vdash \texttt{[]} : \tau \texttt{ list}} \tag{T-Nil}$$

$$\frac{\Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau \texttt{ list}}{\Gamma \vdash e_1 \texttt{ :: } e_2 : \tau \texttt{ list}} \tag{T-Cons}$$

$$\frac{\Gamma \vdash e_1 : \tau' \texttt{ list} \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma, x\colon \tau', y\colon \tau' \texttt{ list} \vdash e_3 : \tau}{\Gamma \vdash \texttt{match } e_1 \texttt{ with [] } \to e_2 \texttt{ | } x \texttt{ :: } y \to e_3 : \tau} \tag{T-Match}$$

EvalContML1

Syntax:

$i \in \mathbf{int}$
$b \in \mathbf{bool}$
$v \in \mathbf{Value} ::= i \mid b$
$e \in \mathbf{Exp} ::= i \mid b \mid e\ op\ e \mid \texttt{if}\ e\ \texttt{then}\ e\ \texttt{else}\ e$
$op \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}$
$k \in \mathbf{Cont} ::= \_ \mid \{\_\ op\ e\} \gg k \mid \{v\ op\ \_\} \gg k \mid \{\texttt{if}\ \_\ \texttt{then}\ e\ \texttt{else}\ e\} \gg k$

継続末尾の _ (とそれに先行する ≫) は省略してよい.

Judgment Form(s):

$$v_1 \Rightarrow k \Downarrow v_2$$

$$e \gg k \Downarrow v$$

$$i_1\ \texttt{plus}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{minus}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{times}\ i_2\ \texttt{is}\ i_3$$

$$i_1\ \texttt{less than}\ i_2\ \texttt{is}\ b_3$$

Derivation Rules:

$$\frac{i \Rightarrow k \Downarrow v}{i \gg k \Downarrow v} \quad \text{(E-Int)}$$

$$\frac{b \Rightarrow k \Downarrow v}{b \gg k \Downarrow v} \quad \text{(E-Bool)}$$

$$\frac{e_1 \gg \{\_\ op\ e_2\} \gg k \Downarrow v}{e_1\ op\ e_2 \gg k \Downarrow v} \quad \text{(E-BinOp)}$$

$$\frac{e_1 \gg \{\texttt{if}\ \_\ \texttt{then}\ e_2\ \texttt{else}\ e_3\} \gg k \Downarrow v}{\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3 \gg k \Downarrow v} \quad \text{(E-If)}$$

$$\frac{}{v \Rightarrow \_ \Downarrow v} \quad \text{(C-Ret)}$$

$$\frac{e \gg \{v_1\ op\ \_\} \gg k \Downarrow v_2}{v_1 \Rightarrow \{\_\ op\ e\} \gg k \Downarrow v_2} \quad \text{(C-EvalR)}$$

$$\frac{i_1\ \texttt{plus}\ i_2\ \texttt{is}\ i_3 \qquad i_3 \Rightarrow k \Downarrow v}{i_2 \Rightarrow \{i_1\ \texttt{+}\ \_\} \gg k \Downarrow v} \quad \text{(C-Plus)}$$

$$\frac{i_1\ \texttt{minus}\ i_2\ \texttt{is}\ i_3 \qquad i_3 \Rightarrow k \Downarrow v}{i_2 \Rightarrow \{i_1\ \texttt{-}\ \_\} \gg k \Downarrow v} \quad \text{(C-Minus)}$$

$$\frac{i_1\ \texttt{times}\ i_2\ \texttt{is}\ i_3 \qquad i_3 \Rightarrow k \Downarrow v}{i_2 \Rightarrow \{i_1\ \texttt{*}\ \_\} \gg k \Downarrow v} \quad \text{(C-Times)}$$

$$\frac{i_1\ \texttt{less than}\ i_2\ \texttt{is}\ b_3 \qquad b_3 \Rightarrow k \Downarrow v}{i_2 \Rightarrow \{i_1\ \texttt{<}\ \_\} \gg k \Downarrow v} \quad \text{(C-Lt)}$$

$$\frac{e_1 \gg k \Downarrow v}{\texttt{true} \Rightarrow \{\texttt{if}\ \_\ \texttt{then}\ e_1\ \texttt{else}\ e_2\} \gg k \Downarrow v} \quad \text{(C-IfT)}$$

$$\frac{e_2 \gg k \Downarrow v}{\texttt{false} \Rightarrow \{\texttt{if}\ \_\ \texttt{then}\ e_1\ \texttt{else}\ e_2\} \gg k \Downarrow v} \quad \text{(C-IfF)}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1\ \texttt{plus}\ i_2\ \texttt{is}\ i_3} \quad \text{(B-Plus)}$$

$$\frac{(i_3 = i_1 - i_2)}{i_1\ \texttt{minus}\ i_2\ \texttt{is}\ i_3} \quad \text{(B-Minus)}$$

$$\frac{(i_3 = i_1 * i_2)}{i_1\ \texttt{times}\ i_2\ \texttt{is}\ i_3} \quad \text{(B-Times)}$$

$$\frac{(b_3 = (i_1 < i_2))}{i_1\ \texttt{less than}\ i_2\ \texttt{is}\ b_3} \quad \text{(B-Lt)}$$

EvalContML4

Syntax:

$i \in \mathbf{int}$
$b \in \mathbf{bool}$
$x, y \in \mathbf{Var}$
$v \in \mathbf{Value} ::= i \mid b \mid (\mathcal{E})[\texttt{fun } x \to e] \mid (\mathcal{E})[\texttt{rec } x \texttt{ = fun } y \to e] \mid [] \mid v :: v \mid [k]$
$\mathcal{E} \in \mathbf{Env} ::= \bullet \mid \mathcal{E}, x = v$
$e \in \mathbf{Exp} ::= i \mid b \mid x \mid e\ op\ e \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid \texttt{let } x \texttt{ = } e \texttt{ in } e$
$\quad \mid \texttt{fun } x \to e \mid e\ e \mid \texttt{let rec } x \texttt{ = fun } y \to e \texttt{ in } e$
$\quad \mid [] \mid e :: e \mid \texttt{match } e \texttt{ with } [] \to e \texttt{ | } x :: y \to e$
$\quad \mid \texttt{letcc } x \texttt{ in } e$
$op \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}$
$k \in \mathbf{Cont} ::= \_ \mid \{\mathcal{E} \vdash \_\ op\ e\} \gg k \mid \{v\ op\ \_\} \gg k \mid \{\mathcal{E} \vdash \texttt{if } \_ \texttt{ then } e \texttt{ else } e\} \gg k$
$\quad \mid \{\mathcal{E} \vdash \texttt{let } x \texttt{ = } \_ \texttt{ in } e\} \gg k \mid \{\mathcal{E} \vdash \_\ e\} \gg k \mid \{v\ \_\} \gg k$
$\quad \mid \{\mathcal{E} \vdash \_ :: e\} \gg k \mid \{v :: \_\} \gg k \mid \{\mathcal{E} \vdash \texttt{match } \_ \texttt{ with } [] \to e \texttt{ | } x :: y \to e\} \gg k$

空の環境 $\bullet$ (とそれに続くコンマ) は入力時には省略する．また，継続末尾の _ (とそれに先行する $\gg$) は省略してよい．

Judgment Form(s):

$$\mathcal{E} \vdash e \gg k \Downarrow v$$

$$v_1 \Rightarrow k \Downarrow v_2$$

$$i_1 \texttt{ plus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ minus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ times } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ less than } i_2 \texttt{ is } b_3$$

Derivation Rules:

$$\frac{i \Rightarrow k \Downarrow v}{\mathcal{E} \vdash i \gg k \Downarrow v} \quad \text{(E-Int)}$$

$$\frac{b \Rightarrow k \Downarrow v}{\mathcal{E} \vdash b \gg k \Downarrow v} \quad \text{(E-Bool)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{if \_ then } e_2 \texttt{ else } e_3\} \gg k \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \gg k \Downarrow v} \quad \text{(E-If)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{\_ } op\ e_2\} \gg k \Downarrow v}{\mathcal{E} \vdash e_1\ op\ e_2 \gg k \Downarrow v} \quad \text{(E-BinOp)}$$

$$\frac{(\mathcal{E}(x) = v_1) \qquad v_1 \Rightarrow k \Downarrow v_2}{\mathcal{E} \vdash x \gg k \Downarrow v_2} \quad \text{(E-Var)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{let } x \texttt{ = \_ in } e_2\} \gg k \Downarrow v}{\mathcal{E} \vdash \texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2 \gg k \Downarrow v} \quad \text{(E-Let)}$$

$$\frac{(\mathcal{E})[\texttt{fun } x \to e] \Rightarrow k \Downarrow v}{\mathcal{E} \vdash \texttt{fun } x \to e \gg k \Downarrow v} \quad \text{(E-Fun)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{\_ } e_2\} \gg k \Downarrow v}{\mathcal{E} \vdash e_1\ e_2 \gg k \Downarrow v} \quad \text{(E-App)}$$

$$\frac{\mathcal{E}, x = (\mathcal{E})[\texttt{rec } x \texttt{ = fun } y \to e_1] \vdash e_2 \gg k \Downarrow v}{\mathcal{E} \vdash \texttt{let rec } x \texttt{ = fun } y \to e_1 \texttt{ in } e_2 \gg k \Downarrow v} \quad \text{(E-LetRec)}$$

$$\frac{[] \Rightarrow k \Downarrow v}{\mathcal{E} \vdash [] \gg k \Downarrow v} \quad \text{(E-Nil)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{\_ :: } e_2\} \gg k \Downarrow v}{\mathcal{E} \vdash e_1 :: e_2 \gg k \Downarrow v} \quad \text{(E-Cons)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{match \_ with } [] \to e_2 \texttt{ | } x :: y \to e_3\} \gg k \Downarrow v}{\mathcal{E} \vdash \texttt{match } e_1 \texttt{ with } [] \to e_2 \texttt{ | } x :: y \to e_3 \gg k \Downarrow v} \quad \text{(E-Match)}$$

$$\frac{\mathcal{E}, x = [k] \vdash e \gg k \Downarrow v}{\mathcal{E} \vdash \texttt{letcc } x \texttt{ in } e \gg k \Downarrow v} \quad \text{(E-LetCc)}$$

$$\frac{}{v \Rightarrow \_ \Downarrow v} \quad \text{(C-Ret)}$$

$$\frac{\mathcal{E} \vdash e \gg \{v_1\ op\ \_\} \gg k \Downarrow v_2}{v_1 \Rightarrow \{\mathcal{E} \vdash \texttt{\_ } op\ e\} \gg k \Downarrow v_2} \quad \text{(C-EvalR)}$$

$$\frac{i_1 \texttt{ plus } i_2 \texttt{ is } i_3 \qquad i_3 \Rightarrow k \Downarrow v}{i_2 \Rightarrow \{i_1 \texttt{ + \_}\} \gg k \Downarrow v} \quad \text{(C-Plus)}$$

$$\frac{i_1 \texttt{ minus } i_2 \texttt{ is } i_3 \qquad i_3 \Rightarrow k \Downarrow v}{i_2 \Rightarrow \{i_1 \texttt{ - \_}\} \gg k \Downarrow v} \tag{C-Minus}$$

$$\frac{i_1 \texttt{ times } i_2 \texttt{ is } i_3 \qquad i_3 \Rightarrow k \Downarrow v}{i_2 \Rightarrow \{i_1 \texttt{ * \_}\} \gg k \Downarrow v} \tag{C-Times}$$

$$\frac{i_1 \texttt{ less than } i_2 \texttt{ is } b_3 \qquad b_3 \Rightarrow k \Downarrow v}{i_2 \Rightarrow \{i_1 \texttt{ < \_}\} \gg k \Downarrow v} \tag{C-Lt}$$

$$\frac{\mathcal{E} \vdash e_1 \gg k \Downarrow v}{\texttt{true} \Rightarrow \{\mathcal{E} \vdash \texttt{if \_ then } e_1 \texttt{ else } e_2\} \gg k \Downarrow v} \tag{C-IfT}$$

$$\frac{\mathcal{E} \vdash e_2 \gg k \Downarrow v}{\texttt{false} \Rightarrow \{\mathcal{E} \vdash \texttt{if \_ then } e_1 \texttt{ else } e_2\} \gg k \Downarrow v} \tag{C-IfF}$$

$$\frac{\mathcal{E}, x = v_1 \vdash e \gg k \Downarrow v_2}{v_1 \Rightarrow \{\mathcal{E} \vdash \texttt{let } x \texttt{ = \_ in } e\} \gg k \Downarrow v_2} \tag{C-LetBody}$$

$$\frac{\mathcal{E} \vdash e \gg \{v_1 \texttt{ \_}\} \gg k \Downarrow v}{v_1 \Rightarrow \{\mathcal{E} \vdash \texttt{\_ } e\} \gg k \Downarrow v} \tag{C-EvalArg}$$

$$\frac{\mathcal{E}, x = v_1 \vdash e \gg k \Downarrow v_2}{v_1 \Rightarrow \{(\mathcal{E})\texttt{[fun } x \texttt{ -> } e\texttt{] \_}\} \gg k \Downarrow v_2} \tag{C-EvalFun}$$

$$\frac{\mathcal{E}, x = (\mathcal{E})\texttt{[rec } x \texttt{ = fun } y \texttt{ -> } e\texttt{]}, y = v_1 \vdash e \gg k \Downarrow v_2}{v_1 \Rightarrow \{(\mathcal{E})\texttt{[rec } x \texttt{ = fun } y \texttt{ -> } e\texttt{] \_}\} \gg k \Downarrow v_2} \tag{C-EvalFunR}$$

$$\frac{v_1 \Rightarrow k_1 \Downarrow v_2}{v_1 \Rightarrow \{\texttt{[}k_1\texttt{] \_}\} \gg k_2 \Downarrow v_2} \tag{C-EvalFunC}$$

$$\frac{\mathcal{E} \vdash e \gg \{v_1 \texttt{ :: \_}\} \gg k \Downarrow v_2}{v_1 \Rightarrow \{\mathcal{E} \vdash \texttt{\_ :: } e\} \gg k \Downarrow v_2} \tag{C-EvalConsR}$$

$$\frac{v_1 \texttt{ :: } v_2 \Rightarrow k \Downarrow v_3}{v_2 \Rightarrow \{v_1 \texttt{ :: \_}\} \gg k \Downarrow v_3} \tag{C-Cons}$$

$$\frac{\mathcal{E} \vdash e_1 \gg k \Downarrow v}{\texttt{[]} \Rightarrow \{\mathcal{E} \vdash \texttt{match \_ with [] -> } e_1 \texttt{ | } x \texttt{ :: } y \texttt{ -> } e_2\} \gg k \Downarrow v} \tag{C-MatchNil}$$

$$\frac{\mathcal{E}, x = v_1, y = v_2 \vdash e_2 \gg k \Downarrow v}{v_1 \texttt{ :: } v_2 \Rightarrow \{\mathcal{E} \vdash \texttt{match \_ with [] -> } e_1 \texttt{ | } x \texttt{ :: } y \texttt{ -> } e_2\} \gg k \Downarrow v} \tag{C-MatchCons}$$

$$\frac{(\boldsymbol{i_3} = \boldsymbol{i_1} + \boldsymbol{i_2})}{\boldsymbol{i_1}\ \texttt{plus}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}} \quad \text{(B-Plus)}$$

$$\frac{(\boldsymbol{i_3} = \boldsymbol{i_1} - \boldsymbol{i_2})}{\boldsymbol{i_1}\ \texttt{minus}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}} \quad \text{(B-Minus)}$$

$$\frac{(\boldsymbol{i_3} = \boldsymbol{i_1} * \boldsymbol{i_2})}{\boldsymbol{i_1}\ \texttt{times}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{i_3}} \quad \text{(B-Times)}$$

$$\frac{(\boldsymbol{b_3} = (\boldsymbol{i_1} < \boldsymbol{i_2}))}{\boldsymbol{i_1}\ \texttt{less than}\ \boldsymbol{i_2}\ \texttt{is}\ \boldsymbol{b_3}} \quad \text{(B-Lt)}$$

EvalDContML4

Syntax:

$$
\begin{array}{l}
i \in \textbf{int} \\
b \in \textbf{bool} \\
x, y \in \textbf{Var} \\
v \in \textbf{Value} ::= i \mid b \mid (\mathcal{E})\texttt{[fun } x \to e\texttt{]} \mid (\mathcal{E})\texttt{[rec } x \texttt{ = fun } y \to e\texttt{]} \mid \texttt{[]} \mid v \texttt{ :: } v \mid \texttt{[}k\texttt{]} \\
\mathcal{E} \in \textbf{Env} ::= \bullet \mid \mathcal{E}, x = v \\
e \in \textbf{Exp} ::= i \mid b \mid x \mid e \; op \; e \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid \texttt{let } x \texttt{ = } e \texttt{ in } e \\
\qquad \mid \texttt{fun } x \to e \mid e\; e \mid \texttt{let rec } x \texttt{ = fun } y \to e \texttt{ in } e \mid \texttt{[]} \\
\qquad \mid e \texttt{ :: } e \mid \texttt{match } e \texttt{ with [] } \to e \texttt{ | } x \texttt{ :: } y \to e \\
\qquad \mid \{\; e \;\} \mid \texttt{shift } x \texttt{ in } e \\
op \in \textbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<} \\
kk \in \textbf{MCont} ::= \texttt{\_\_} \mid k \ggg kk \\
k \in \textbf{Cont} ::= \texttt{\_} \mid \{\mathcal{E} \vdash \texttt{\_ } op\; e\} \gg k \mid \{v \; op \texttt{ \_}\} \gg k \mid \{\mathcal{E} \vdash \texttt{if \_ then } e \texttt{ else } e\} \gg k \\
\qquad \mid \{\mathcal{E} \vdash \texttt{let } x \texttt{ = \_ in } e\} \gg k \mid \{\mathcal{E} \vdash \texttt{\_ } e\} \gg k \mid \{v \texttt{ \_}\} \gg k \mid \{\mathcal{E} \vdash \texttt{\_ :: } e\} \gg k \\
\qquad \mid \{v \texttt{ :: \_}\} \gg k \mid \{\mathcal{E} \vdash \texttt{match \_ with [] } \to e \texttt{ | } x \texttt{ :: } y \to e\} \gg k
\end{array}
$$

Judgment Form(s):

$$v_1 \Rightarrow k \ggg kk \Downarrow v_2$$

$$\mathcal{E} \vdash e \gg k \ggg kk \Downarrow v$$

$$i_1 \texttt{ plus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ minus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ times } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ less than } i_2 \texttt{ is } b_3$$

Derivation Rules:

$$\frac{i \Rightarrow k \ggg kk \Downarrow v}{\mathcal{E} \vdash i \gg k \ggg kk \Downarrow v} \quad \text{(E-Int)}$$

$$\frac{b \Rightarrow k \ggg kk \Downarrow v}{\mathcal{E} \vdash b \gg k \ggg kk \Downarrow v} \quad \text{(E-Bool)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{if \_ then } e_2 \texttt{ else } e_3\} \gg k \ggg kk \Downarrow v}{\mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \gg k \ggg kk \Downarrow v} \quad \text{(E-If)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \_\ op\ e_2\} \gg k \ggg kk \Downarrow v}{\mathcal{E} \vdash e_1\ op\ e_2 \gg k \ggg kk \Downarrow v} \quad \text{(E-BinOp)}$$

$$\frac{(\mathcal{E}(x) = v_1) \qquad v_1 \Rightarrow k \ggg kk \Downarrow v_2}{\mathcal{E} \vdash x \gg k \ggg kk \Downarrow v_2} \quad \text{(E-Var)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{let}\ x\ \texttt{=}\ \_\ \texttt{in}\ e_2\} \gg k \ggg kk \Downarrow v}{\mathcal{E} \vdash \texttt{let}\ x\ \texttt{=}\ e_1\ \texttt{in}\ e_2 \gg k \ggg kk \Downarrow v} \quad \text{(E-Let)}$$

$$\frac{(\mathcal{E})[\texttt{fun}\ x \to e] \Rightarrow k \ggg kk \Downarrow v}{\mathcal{E} \vdash \texttt{fun}\ x \to e \gg k \ggg kk \Downarrow v} \quad \text{(E-Fun)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \_\ e_2\} \gg k \ggg kk \Downarrow v}{\mathcal{E} \vdash e_1\ e_2 \gg k \ggg kk \Downarrow v} \quad \text{(E-App)}$$

$$\frac{\mathcal{E}, x = (\mathcal{E})[\texttt{rec}\ x\ \texttt{=}\ \texttt{fun}\ y \to e_1] \vdash e_2 \gg k \ggg kk \Downarrow v}{\mathcal{E} \vdash \texttt{let rec}\ x\ \texttt{=}\ \texttt{fun}\ y \to e_1\ \texttt{in}\ e_2 \gg k \ggg kk \Downarrow v} \quad \text{(E-LetRec)}$$

$$\frac{[] \Rightarrow k \ggg kk \Downarrow v}{\mathcal{E} \vdash [] \gg k \ggg kk \Downarrow v} \quad \text{(E-Nil)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \_ :: e_2\} \gg k \ggg kk \Downarrow v}{\mathcal{E} \vdash e_1 :: e_2 \gg k \ggg kk \Downarrow v} \quad \text{(E-Cons)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg \{\mathcal{E} \vdash \texttt{match}\ \_\ \texttt{with}\ [] \to e_2\ \texttt{|}\ x :: y \to e_3\} \gg k \ggg kk \Downarrow v}{\mathcal{E} \vdash \texttt{match}\ e_1\ \texttt{with}\ [] \to e_2\ \texttt{|}\ x :: y \to e_3 \gg k \ggg kk \Downarrow v} \quad \text{(E-Match)}$$

$$\frac{\mathcal{E} \vdash e \gg \_ \ggg k \ggg kk \Downarrow v}{\mathcal{E} \vdash \{\ e\ \} \gg k \ggg kk \Downarrow v} \quad \text{(E-Reset)}$$

$$\frac{\mathcal{E}, x = [k] \vdash e \gg \_ \ggg kk \Downarrow v}{\mathcal{E} \vdash \texttt{shift}\ x\ \texttt{in}\ e \gg k \ggg kk \Downarrow v} \quad \text{(E-Shift)}$$

$$\frac{}{v \Rightarrow \_ \ggg \_\_ \Downarrow v} \quad \text{(C-RetRet)}$$

$$\frac{v_1 \Rightarrow k \ggg kk \Downarrow v_2}{v_1 \Rightarrow \_ \ggg k \ggg kk \Downarrow v_2} \quad \text{(C-RetCont)}$$

$$\frac{\mathcal{E} \vdash e \gg \{v_1\ op\ \_\} \gg k \ggg kk \Downarrow v_2}{v_1 \Rightarrow \{\mathcal{E} \vdash \_\ op\ e\} \gg k \ggg kk \Downarrow v_2} \quad \text{(C-EvalR)}$$

$$\frac{i_1 \texttt{ plus } i_2 \texttt{ is } i_3 \qquad i_3 \Rightarrow k \ggg kk \Downarrow v}{i_2 \Rightarrow \{i_1 \texttt{ + \_}\} \gg k \ggg kk \Downarrow v} \quad \text{(C-Plus)}$$

$$\frac{i_1 \texttt{ minus } i_2 \texttt{ is } i_3 \qquad i_3 \Rightarrow k \ggg kk \Downarrow v}{i_2 \Rightarrow \{i_1 \texttt{ - \_}\} \gg k \ggg kk \Downarrow v} \quad \text{(C-Minus)}$$

$$\frac{i_1 \texttt{ times } i_2 \texttt{ is } i_3 \qquad i_3 \Rightarrow k \ggg kk \Downarrow v}{i_2 \Rightarrow \{i_1 \texttt{ * \_}\} \gg k \ggg kk \Downarrow v} \quad \text{(C-Times)}$$

$$\frac{i_1 \texttt{ less than } i_2 \texttt{ is } b_3 \qquad b_3 \Rightarrow k \ggg kk \Downarrow v}{i_2 \Rightarrow \{i_1 \texttt{ < \_}\} \gg k \ggg kk \Downarrow v} \quad \text{(C-Lt)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg k \ggg kk \Downarrow v}{\texttt{true} \Rightarrow \{\mathcal{E} \vdash \texttt{if \_ then } e_1 \texttt{ else } e_2\} \gg k \ggg kk \Downarrow v} \quad \text{(C-IfT)}$$

$$\frac{\mathcal{E} \vdash e_2 \gg k \ggg kk \Downarrow v}{\texttt{false} \Rightarrow \{\mathcal{E} \vdash \texttt{if \_ then } e_1 \texttt{ else } e_2\} \gg k \ggg kk \Downarrow v} \quad \text{(C-IfF)}$$

$$\frac{\mathcal{E}, x \texttt{ = } v_1 \vdash e \gg k \ggg kk \Downarrow v_2}{v_1 \Rightarrow \{\mathcal{E} \vdash \texttt{let } x \texttt{ = \_ in } e\} \gg k \ggg kk \Downarrow v_2} \quad \text{(C-LetBody)}$$

$$\frac{\mathcal{E} \vdash e \gg \{v_1 \texttt{ \_}\} \gg k \ggg kk \Downarrow v}{v_1 \Rightarrow \{\mathcal{E} \vdash \texttt{ \_ } e\} \gg k \ggg kk \Downarrow v} \quad \text{(C-EvalArg)}$$

$$\frac{\mathcal{E}, x \texttt{ = } v_1 \vdash e \gg k \ggg kk \Downarrow v_2}{v_1 \Rightarrow \{\texttt{(}\mathcal{E}\texttt{)[fun } x \to e\texttt{] \_}\} \gg k \ggg kk \Downarrow v_2} \quad \text{(C-EvalFun)}$$

$$\frac{\mathcal{E}, x \texttt{ = (}\mathcal{E}\texttt{)[rec } x \texttt{ = fun } y \to e\texttt{]}, y \texttt{ = } v_1 \vdash e \gg k \ggg kk \Downarrow v_2}{v_1 \Rightarrow \{\texttt{(}\mathcal{E}\texttt{)[rec } x \texttt{ = fun } y \to e\texttt{] \_}\} \gg k \ggg kk \Downarrow v_2} \quad \text{(C-EvalFunR)}$$

$$\frac{v_1 \Rightarrow k_1 \ggg k_2 \ggg kk \Downarrow v_2}{v_1 \Rightarrow \{\texttt{[}k_1\texttt{] \_}\} \gg k_2 \ggg kk \Downarrow v_2} \quad \text{(C-EvalFunC)}$$

$$\frac{\mathcal{E} \vdash e \gg \{v_1 \texttt{ :: \_}\} \gg k \ggg kk \Downarrow v_2}{v_1 \Rightarrow \{\mathcal{E} \vdash \texttt{ \_ :: } e\} \gg k \ggg kk \Downarrow v_2} \quad \text{(C-EvalConsR)}$$

$$\frac{v_1 \texttt{ :: } v_2 \Rightarrow k \ggg kk \Downarrow v_3}{v_2 \Rightarrow \{v_1 \texttt{ :: \_}\} \gg k \ggg kk \Downarrow v_3} \quad \text{(C-Cons)}$$

$$\frac{\mathcal{E} \vdash e_1 \gg k \ggg kk \Downarrow v}{\texttt{[]} \Rightarrow \{\mathcal{E} \vdash \texttt{match \_ with [] } \to e_1 \texttt{ | } x \texttt{ :: } y \to e_2\} \gg k \ggg kk \Downarrow v} \quad \text{(C-MatchNil)}$$

$$\frac{\boldsymbol{\mathcal{E}}, \boldsymbol{x} = \boldsymbol{v_1}, \boldsymbol{y} = \boldsymbol{v_2} \vdash \boldsymbol{e_2} \gg \boldsymbol{k} \ggg \boldsymbol{kk} \Downarrow \boldsymbol{v}}{\boldsymbol{v_1} :: \boldsymbol{v_2} \Rightarrow \{\boldsymbol{\mathcal{E}} \vdash \texttt{match \_ with []} \rightarrow \boldsymbol{e_1} \texttt{ | } \boldsymbol{x} :: \boldsymbol{y} \rightarrow \boldsymbol{e_2}\} \gg \boldsymbol{k} \ggg \boldsymbol{kk} \Downarrow \boldsymbol{v}} \quad \text{(C-MatchCons)}$$

$$\frac{(\boldsymbol{i_3} = \boldsymbol{i_1} + \boldsymbol{i_2})}{\boldsymbol{i_1} \texttt{ plus } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}} \quad \text{(B-Plus)}$$

$$\frac{(\boldsymbol{i_3} = \boldsymbol{i_1} - \boldsymbol{i_2})}{\boldsymbol{i_1} \texttt{ minus } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}} \quad \text{(B-Minus)}$$

$$\frac{(\boldsymbol{i_3} = \boldsymbol{i_1} * \boldsymbol{i_2})}{\boldsymbol{i_1} \texttt{ times } \boldsymbol{i_2} \texttt{ is } \boldsymbol{i_3}} \quad \text{(B-Times)}$$

$$\frac{(\boldsymbol{b_3} = (\boldsymbol{i_1} < \boldsymbol{i_2}))}{\boldsymbol{i_1} \texttt{ less than } \boldsymbol{i_2} \texttt{ is } \boldsymbol{b_3}} \quad \text{(B-Lt)}$$

EvalRefML3

Syntax:

$$
\begin{array}{l}
i \in \mathbf{int} \\
b \in \mathbf{bool} \\
x, y \in \mathbf{Var} \\
l \in \mathbf{Loc} \\
v \in \mathbf{Value} ::= i \mid b \mid l \mid (\mathcal{E})[\texttt{fun } x \to e] \mid (\mathcal{E})[\texttt{rec } x \texttt{ = fun } y \to e] \\
\mathcal{E} \in \mathbf{Env} ::= \bullet \mid \mathcal{E}, x \texttt{ = } v \\
S \in \mathbf{Store} ::= \bullet \mid S, l \texttt{ = } v \\
e \in \mathbf{Exp} ::= i \mid b \mid x \mid e \; op \; e \mid \texttt{if } e \texttt{ then } e \texttt{ else } e \mid \texttt{let } x \texttt{ = } e \texttt{ in } e \\
\qquad\qquad \mid \texttt{fun } x \to e \mid e \; e \mid \texttt{let rec } x \texttt{ = fun } y \to e \texttt{ in } e \\
\qquad\qquad \mid \texttt{ref } e \mid \texttt{!}e \mid e \texttt{ := } e \\
op \in \mathbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{<}
\end{array}
$$

空の環境やストア $\bullet$ (とそれに続くコンマ) は入力時には省略する．また，ストアが空の場合，判断中でそれに先行する，もしくは続く / も省略してよい．また，E-Assign に現れる記法 $S[l = v]$ は，S に現れる $l = \ldots$ を $l = v$ で置き換えたようなストアであり，正確には以下のように定義される．

$$
\begin{array}{rcl}
(S, l \texttt{ = } v)[l = v'] & = & S, l \texttt{ = } v' \\
(S, l \texttt{ = } v)[l' = v'] & = & (S[l' = v']), l \texttt{ = } v \qquad (\text{if } l \neq l')
\end{array}
$$

Judgment Form(s):

$$S_1 \texttt{ / } \mathcal{E} \vdash e \Downarrow v \texttt{ / } S_2$$

$$i_1 \texttt{ plus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ minus } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ times } i_2 \texttt{ is } i_3$$

$$i_1 \texttt{ less than } i_2 \texttt{ is } b_3$$

Derivation Rules:

$$\frac{}{S \texttt{ / } \mathcal{E} \vdash i \Downarrow i \texttt{ / } S} \quad \text{(E-Int)}$$

$$\frac{}{S \texttt{ / } \mathcal{E} \vdash b \Downarrow b \texttt{ / } S} \quad \text{(E-Bool)}$$

$$\frac{S_1 \texttt{ / } \mathcal{E} \vdash e_1 \Downarrow \texttt{true} \texttt{ / } S_2 \qquad S_2 \texttt{ / } \mathcal{E} \vdash e_2 \Downarrow v \texttt{ / } S_3}{S_1 \texttt{ / } \mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v \texttt{ / } S_3} \quad \text{(E-IfT)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow \texttt{false} \ / \ S_2 \qquad S_2 \ / \ \mathcal{E} \vdash e_3 \Downarrow v \ / \ S_3}{S_1 \ / \ \mathcal{E} \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow v \ / \ S_3} \quad \text{(E-IfF)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow i_1 \ / \ S_2 \qquad S_2 \ / \ \mathcal{E} \vdash e_2 \Downarrow i_2 \ / \ S_3 \qquad i_1 \texttt{ plus } i_2 \texttt{ is } i_3}{S_1 \ / \ \mathcal{E} \vdash e_1 + e_2 \Downarrow i_3 \ / \ S_3} \quad \text{(E-Plus)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow i_1 \ / \ S_2 \qquad S_2 \ / \ \mathcal{E} \vdash e_2 \Downarrow i_2 \ / \ S_3 \qquad i_1 \texttt{ minus } i_2 \texttt{ is } i_3}{S_1 \ / \ \mathcal{E} \vdash e_1 - e_2 \Downarrow i_3 \ / \ S_3} \quad \text{(E-Minus)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow i_1 \ / \ S_2 \qquad S_2 \ / \ \mathcal{E} \vdash e_2 \Downarrow i_2 \ / \ S_3 \qquad i_1 \texttt{ times } i_2 \texttt{ is } i_3}{S_1 \ / \ \mathcal{E} \vdash e_1 * e_2 \Downarrow i_3 \ / \ S_2} \quad \text{(E-Times)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow i_1 \ / \ S_2 \qquad S_2 \ / \ \mathcal{E} \vdash e_2 \Downarrow i_2 \ / \ S_3 \qquad i_1 \texttt{ less than } i_2 \texttt{ is } b_3}{S_1 \ / \ \mathcal{E} \vdash e_1 < e_2 \Downarrow b_3 \ / \ S_3} \quad \text{(E-Lt)}$$

$$\frac{(\mathcal{E}(x) = v)}{S \ / \ \mathcal{E} \vdash x \Downarrow v \ / \ S} \quad \text{(E-Var)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow v_1 \ / \ S_2 \qquad S_2 \ / \ \mathcal{E}, x = v_1 \vdash e_2 \Downarrow v \ / \ S_3}{S_1 \ / \ \mathcal{E} \vdash \texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2 \Downarrow v \ / \ S_3} \quad \text{(E-Let)}$$

$$\frac{}{S \ / \ \mathcal{E} \vdash \texttt{fun } x \to e \Downarrow (\mathcal{E})\texttt{[fun } x \to e\texttt{]} \ / \ S} \quad \text{(E-Fun)}$$

$$\frac{\begin{array}{c} S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)\texttt{[fun } x \to e_0\texttt{]} \ / \ S_2 \qquad S_2 \ / \ \mathcal{E} \vdash e_2 \Downarrow v_2 \ / \ S_3 \\ S_3 \ / \ \mathcal{E}_2, x = v_2 \vdash e_0 \Downarrow v \ / \ S_4 \end{array}}{S_1 \ / \ \mathcal{E} \vdash e_1 \ e_2 \Downarrow v \ / \ S_4} \quad \text{(E-App)}$$

$$\frac{S_1 \ / \ \mathcal{E}, x = (\mathcal{E})\texttt{[rec } x \texttt{ = fun } y \to e_1\texttt{]} \vdash e_2 \Downarrow v \ / \ S_2}{S_1 \ / \ \mathcal{E} \vdash \texttt{let rec } x \texttt{ = fun } y \to e_1 \texttt{ in } e_2 \Downarrow v \ / \ S_2} \quad \text{(E-LetRec)}$$

$$\frac{\begin{array}{c} S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)\texttt{[rec } x \texttt{ = fun } y \to e_0\texttt{]} \ / \ S_2 \qquad S_2 \ / \ \mathcal{E} \vdash e_2 \Downarrow v_2 \ / \ S_3 \\ S_3 \ / \ \mathcal{E}_2, x = (\mathcal{E}_2)\texttt{[rec } x \texttt{ = fun } y \to e_0\texttt{]}, y = v_2 \vdash e_0 \Downarrow v \ / \ S_4 \end{array}}{S_1 \ / \ \mathcal{E} \vdash e_1 \ e_2 \Downarrow v \ / \ S_4} \quad \text{(E-AppRec)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e \Downarrow v \ / \ S_2 \qquad (l \notin dom(S_2))}{S_1 \ / \ \mathcal{E} \vdash \texttt{ref } e \Downarrow l \ / \ S_2, l = v} \quad \text{(E-Ref)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e \Downarrow l \ / \ S_2 \qquad (S_2(l) = v)}{S_1 \ / \ \mathcal{E} \vdash \texttt{!}e \Downarrow v \ / \ S_2} \quad \text{(E-Deref)}$$

$$\frac{S_1 \ / \ \mathcal{E} \vdash e_1 \Downarrow l \ / \ S_2 \qquad S_2 \ / \ \mathcal{E} \vdash e_2 \Downarrow v \ / \ S_3 \qquad (S_4 = S_3[l = v])}{S_1 \ / \ \mathcal{E} \vdash e_1 \ \texttt{:=} \ e_2 \Downarrow v \ / \ S_4} \quad \text{(E-Assign)}$$

$$\frac{(i_3 = i_1 + i_2)}{i_1 \ \texttt{plus} \ i_2 \ \texttt{is} \ i_3} \quad \text{(B-Plus)}$$

$$\frac{(i_3 = i_1 - i_2)}{i_1 \ \texttt{minus} \ i_2 \ \texttt{is} \ i_3} \quad \text{(B-Minus)}$$

$$\frac{(i_3 = i_1 * i_2)}{i_1 \ \texttt{times} \ i_2 \ \texttt{is} \ i_3} \quad \text{(B-Times)}$$

$$\frac{(b_3 = (i_1 < i_2))}{i_1 \ \texttt{less than} \ i_2 \ \texttt{is} \ b_3} \quad \text{(B-Lt)}$$

While

Syntax:

$$
\begin{array}{l}
i \in \textbf{int} \\
bv \in \textbf{bool} \\
x, y \in \textbf{Var} \\
\sigma \in \textbf{Store} ::= \bullet \mid \sigma, x \texttt{ = } i \\
a \in \textbf{AExp} ::= i \mid x \mid a \; aop \; a \\
aop \in \textbf{Prim} ::= \texttt{+} \mid \texttt{-} \mid \texttt{*} \\
b \in \textbf{BExp} ::= bv \mid \texttt{!}b \mid b \; lop \; b \mid a \; comp \; a \\
lop \in \textbf{LOp} ::= \texttt{\&\&} \mid \texttt{||} \\
comp \in \textbf{Comp} ::= \texttt{<} \mid \texttt{=} \mid \texttt{<=} \\
c \in \textbf{Com} ::= \texttt{skip} \mid x \texttt{:=} a \mid c \texttt{;} c \mid \texttt{if } b \texttt{ then } c \texttt{ else } c \mid \texttt{while (}b\texttt{) do } c
\end{array}
$$

Judgment Form(s):

$$\sigma \vdash a \Downarrow i$$

$$\sigma \vdash b \Downarrow bv$$

$$c \texttt{ changes } \sigma_1 \texttt{ to } \sigma_2$$

Derivation Rules:

$$\frac{}{\sigma \vdash i \Downarrow i} \quad \text{(A-Const)}$$

$$\frac{(\sigma(x) = i)}{\sigma \vdash x \Downarrow i} \quad \text{(A-Var)}$$

$$\frac{\sigma \vdash a_1 \Downarrow i_1 \qquad \sigma \vdash a_2 \Downarrow i_2 \qquad (i_3 = i_1 + i_2)}{\sigma \vdash a_1 \texttt{ + } a_2 \Downarrow i_3} \quad \text{(A-Plus)}$$

$$\frac{\sigma \vdash a_1 \Downarrow i_1 \qquad \sigma \vdash a_2 \Downarrow i_2 \qquad (i_3 = i_1 - i_2)}{\sigma \vdash a_1 \texttt{ - } a_2 \Downarrow i_3} \quad \text{(A-Minus)}$$

$$\frac{\sigma \vdash a_1 \Downarrow i_1 \qquad \sigma \vdash a_2 \Downarrow i_2 \qquad (i_3 = i_1 * i_2)}{\sigma \vdash a_1 \texttt{ * } a_2 \Downarrow i_3} \quad \text{(A-Times)}$$

$$\frac{}{\sigma \vdash bv \Downarrow bv} \quad \text{(B-Const)}$$

$$\frac{\boldsymbol{\sigma} \vdash \boldsymbol{b} \Downarrow \boldsymbol{bv_1} \qquad (\boldsymbol{bv_2} = \neg \boldsymbol{bv_1})}{\boldsymbol{\sigma} \vdash \texttt{!}\boldsymbol{b} \Downarrow \boldsymbol{bv_2}} \tag{B-Not}$$

$$\frac{\boldsymbol{\sigma} \vdash \boldsymbol{b_1} \Downarrow \boldsymbol{bv_1} \qquad \boldsymbol{\sigma} \vdash \boldsymbol{b_2} \Downarrow \boldsymbol{bv_2} \qquad (\boldsymbol{bv_3} = (\boldsymbol{bv_1} \wedge \boldsymbol{bv_2}))}{\boldsymbol{\sigma} \vdash \boldsymbol{b_1} \texttt{ \&\& } \boldsymbol{b_2} \Downarrow \boldsymbol{bv_3}} \tag{B-And}$$

$$\frac{\boldsymbol{\sigma} \vdash \boldsymbol{b_1} \Downarrow \boldsymbol{bv_1} \qquad \boldsymbol{\sigma} \vdash \boldsymbol{b_2} \Downarrow \boldsymbol{bv_2} \qquad (\boldsymbol{bv_3} = (\boldsymbol{bv_1} \vee \boldsymbol{bv_2}))}{\boldsymbol{\sigma} \vdash \boldsymbol{b_1} \texttt{ || } \boldsymbol{b_2} \Downarrow \boldsymbol{bv_3}} \tag{B-Or}$$

$$\frac{\boldsymbol{\sigma} \vdash \boldsymbol{a_1} \Downarrow \boldsymbol{i_1} \qquad \boldsymbol{\sigma} \vdash \boldsymbol{a_2} \Downarrow \boldsymbol{i_2} \qquad (\boldsymbol{bv} = (\boldsymbol{i_1} < \boldsymbol{i_2}))}{\boldsymbol{\sigma} \vdash \boldsymbol{a_1} \texttt{ < } \boldsymbol{a_2} \Downarrow \boldsymbol{bv}} \tag{B-Lt}$$

$$\frac{\boldsymbol{\sigma} \vdash \boldsymbol{a_1} \Downarrow \boldsymbol{i_1} \qquad \boldsymbol{\sigma} \vdash \boldsymbol{a_2} \Downarrow \boldsymbol{i_2} \qquad (\boldsymbol{bv} = (\boldsymbol{i_1} = \boldsymbol{i_2}))}{\boldsymbol{\sigma} \vdash \boldsymbol{a_1} \texttt{ = } \boldsymbol{a_2} \Downarrow \boldsymbol{bv}} \tag{B-Eq}$$

$$\frac{\boldsymbol{\sigma} \vdash \boldsymbol{a_1} \Downarrow \boldsymbol{i_1} \qquad \boldsymbol{\sigma} \vdash \boldsymbol{a_2} \Downarrow \boldsymbol{i_2} \qquad (\boldsymbol{bv_3} = (\boldsymbol{i_1} \leq \boldsymbol{i_2}))}{\boldsymbol{\sigma} \vdash \boldsymbol{a_1} \texttt{ <= } \boldsymbol{a_2} \Downarrow \boldsymbol{bv}} \tag{B-Le}$$

$$\frac{}{\texttt{skip changes } \boldsymbol{\sigma} \texttt{ to } \boldsymbol{\sigma}} \tag{C-Skip}$$

$$\frac{\boldsymbol{\sigma_1} \vdash \boldsymbol{a} \Downarrow \boldsymbol{i} \qquad (\boldsymbol{\sigma_2} = \boldsymbol{\sigma_1}[\boldsymbol{i}/\boldsymbol{x}])}{\boldsymbol{x}\texttt{:=}\boldsymbol{a} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_2}} \tag{C-Assign}$$

$$\frac{\boldsymbol{c_1} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_2} \qquad \boldsymbol{c_2} \texttt{ changes } \boldsymbol{\sigma_2} \texttt{ to } \boldsymbol{\sigma_3}}{\boldsymbol{c_1}\texttt{;}\boldsymbol{c_2} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_3}} \tag{C-Seq}$$

$$\frac{\boldsymbol{\sigma_1} \vdash \boldsymbol{b} \Downarrow \texttt{true} \qquad \boldsymbol{c_1} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_2}}{\texttt{if } \boldsymbol{b} \texttt{ then } \boldsymbol{c_1} \texttt{ else } \boldsymbol{c_2} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_2}} \tag{C-IfT}$$

$$\frac{\boldsymbol{\sigma_1} \vdash \boldsymbol{b} \Downarrow \texttt{false} \qquad \boldsymbol{c_2} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_2}}{\texttt{if } \boldsymbol{b} \texttt{ then } \boldsymbol{c_1} \texttt{ else } \boldsymbol{c_2} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_2}} \tag{C-IfF}$$

$$\frac{\boldsymbol{\sigma_1} \vdash \boldsymbol{b} \Downarrow \texttt{true} \qquad \boldsymbol{c} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_2} \qquad \texttt{while (}\boldsymbol{b}\texttt{) do } \boldsymbol{c} \texttt{ changes } \boldsymbol{\sigma_2} \texttt{ to } \boldsymbol{\sigma_3}}{\texttt{while (}\boldsymbol{b}\texttt{) do } \boldsymbol{c} \texttt{ changes } \boldsymbol{\sigma_1} \texttt{ to } \boldsymbol{\sigma_3}} \tag{C-WhileT}$$

$$\frac{\boldsymbol{\sigma} \vdash \boldsymbol{b} \Downarrow \texttt{false}}{\texttt{while (}\boldsymbol{b}\texttt{) do } \boldsymbol{c} \texttt{ changes } \boldsymbol{\sigma} \texttt{ to } \boldsymbol{\sigma}} \tag{C-WhileF}$$