Gradual Typing for Generics

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Abstract

Gradual typing is a framework to combine static and dynamic typing in a single programming language. In this paper, we develop a gradual type system for class-based object-oriented languages with generics. We introduce a special type to denote dynamically typed parts of a program; unlike dynamic types introduced to C♯ 4.0, however, our type system allows for more seamless integration of dynamically and statically typed code.

We formalize a gradual type system for Featherweight GJ with a semantics given by a translation that inserts explicit run-time checks. The type system guarantees that statically typed parts of a program do not go wrong, even if it includes dynamically typed parts. We also describe a basic implementation scheme for Java and report preliminary performance evaluation.

1. Introduction

Statically and dynamically typed languages have their own benefits. On the one hand, statically typed languages enjoy type safety properties; on the other hand, dynamically typed languages are said to be suitable for rapid prototyping. There is a significant amount of work (e.g., [1, 3, 5, 7, 9, 11, 14, 15, 25–27] to cite some) to integrate both kinds of languages to have the best of both worlds. Siek and Taha have coined the term "gradual typing" [25] for a particular style of linguistic support of the seamless integration of static and dynamic typing in a single language. A typical gradual type system introduces to a statically typed language a special type (often called `dynamic`) to specify dynamically typed parts in a program and allows a program to be partially typed, or even fully dynamically typed.

One of the main challenges in the design of a gradual type system is to give a flexible type compatibility relation, which is an extension of subtyping and used for assignments and argument passing. For example, a gradual type system usually assumes `dynamic` to be compatible with any type so that a statically typed expression to be used where `dynamic` is expected and vice versa. Moreover, when types have structures (as in function types), the compatibility relation usually allows structural comparison: for example, a function type, say `dynamic` → `int`, is compatible with `int` → `int` [25], which is useful in higher-order programs.

The other, more technical challenge is to establish some safety property even for partially typed programs. In fact, it is possible to ensure that run-time errors are always due to a dynamically typed part in a program. Roughly speaking, the main idea is to insert run-time checks between the "border" between the statically and dynamically typed worlds to prevent statically typed code from going wrong. A key idea here is that the insertion can be guided by the use of the compatibility relation.

In this paper, we develop a gradual type system for class-based object-oriented languages with generics. Although there are similar attempts at mixing static and dynamic typing in object-oriented languages [3, 5, 18, 26, 35], (to our knowledge) very few take generics into account. One notable exception is dynamic types for C♯ 4.0 [4], but the integration of dynamic and static typing is not as smooth as one might expect. For example, it requires tedious coding to convert a collection whose element type is statically known, say, to be integers to a collection of dynamically typed values. We design a flexible compatibility relation, which allows, for example, `List<Integer>` to be used as `List<dynamic>` and vice versa. Since the type system has inheritance-based subtyping, it is not a trivial task to give a reasonable compatibility relation. We also introduce the notion of *bounded dynamic types*, which have characteristics of both dynamic and static types, to mediate bounded polymorphism and dynamic typing. Based on these ideas, we formalize FGJ$^{\texttt{dyn}}$, an extension of Featherweight GJ (FGJ) [16] with bounded dynamic types and prove the desired safety property, which states that statically typed parts in a program cannot go wrong. In particular, it implies the standard type safety for a program that does not contain any dynamic types. The semantics of FGJ$^{\texttt{dyn}}$—the surface language in which programs are written—is given by a translation to an intermediate language FGJ$^{(\!(\cdot)\!)}$, in which run-time checks are explicit. The

translation is not only expediency for the formal proof but also a guide to implementation.

Our main contributions can be summarized as follows:

- A flexible compatibility relation for parametric types;
- The introduction of bounded dynamic types;
- Formalization of the language with generics and dynamic types; and
- Proof of safety properties, which show that statically typed parts in a program never go wrong.

We are currently developing a compiler for gradually typed Java. We also describe our basic implementation scheme. This work at an earlier stage has been reported at the STOP'09 workshop [19], where we have only sketched the combination of generics and dynamic typing and its formalization. In this paper, we have revised the formal definition of both surface and intermediate languages significantly and proved safety properties.

The rest of the paper is organized as follows. Section 2 gives an overview of gradual typing for a class-based language with generics. Then, Sections 3 and 4 give the formalization of our proposal and prove desired properties. Section 5 describes our implementation scheme for Java and reports very preliminary benchmark results. After Section 6 discusses related work, Section 7 gives concluding remarks.

2. Gradual Typing for Generics

Following the previous approaches to gradual typing [25, 26], we introduce a special type (called `dyn` in this paper) that represents dynamically typed portions in a program to a class-based language with generics. A variable can be declared to have the dynamic type; then, any expressions can be assigned to it and the variable can be used as an expression of any type. In this section, we first describe how `dyn` interacts with generics by means of examples and then what kind of dynamic checks are performed to prevent statically typed parts from going wrong.

We use the following simple generic class as a running example:

```
class Cell<X> {
  X x;  Cell(X x){ this.x=x; }
  void set(X x){ this.x=x; }
}
```

`Cell` is a class of one-element containers, where the element type is parameterized as `X`. The element is accessed through the field `x` and modified through method `set`. We will also use classes `Shape` and its subclasses `Rectangle` and `Polygon`. (Neither `Rectangle` nor `Polygon` extends the other.) Moreover, class `Shape` has method with the signature

```
boolean contains(double x, double y);
```

which returns whether a given point at $(\mathtt{x}, \mathtt{y})$ is inside the shape.

2.1 Type `dyn` as Type Arguments

One natural consequence of the introduction of `dyn` as a type is that `dyn` can be used as a type argument to a generic class. For example, a programmer can use a variable `c1` of type `Cell<dyn>`. This type is similar to `Cell<Object>` in the sense that one can set anything to it.

```
Cell<dyn> c1 = ...;
c1.set(new Polygon(...));
c1.set(new Integer(1));
```

Unlike `Cell<Object>`, however, the type of field `x` is `dyn`, which represents dynamically typed code and accepts any method invocation and field access

```
dyn fld = c1.x.anyField;
dyn ret = c1.x.anyMethod(...);
```

which are assumed to return `dyn`. Also, `dyn` can be assigned to any variable.

```
boolean b = c1.x.contains(1,1);
            // The type of RHS is dyn
```

Of course, it must be checked at run time whether these fields and methods really exists and whether an assignment is valid.

In the previous work on gradual typing for a language with subtyping, the subtyping relation is replaced with the compatibility relation [25, 26], which, for example, allows statically typed expressions to be passed to where `dyn` is expected and vice versa. The compatibility relation should be rich enough to support flexible integration of statically and dynamically typed code and, for type systems with structural subtyping, its definition requires careful examination.

We introduce a rich compatibility relation for parametric types. In particular, we allow an expression of a `dyn`-free type, say `Cell<Rectangle>`, to be assigned to a variable whose type involves `dyn` as a type argument, say `Cell<dyn>`. For example, the following code is accepted by the type system:

```
Cell<dyn> c1
  = new Cell<Rectangle>(new Rectangle(...));
```

Note that `c1` will point to an object that can store only `Rectangles`, rather than anything (as indicated by `dyn`). So, actually, the invocation of `set` should check at run time whether the actual argument is a valid one.

```
c1.set( new Rectangle(...) ); // succeeds
c1.set( new Polygon(...) );   // fails
```

The intuition behind a parametric type to which `dyn` is given as an argument is that it denotes the set of types where `dyn` is replaced with any type. For example, a variable of type `Cell<dyn>` may point to objects `new Cell<Shape>()`, `new Cell<Rectangle>()`, `new Cell<Integer>()`, and so on. In this sense, type `Cell<dyn>` is closer to the wildcard type `Cell<?>` than `Cell<Object>`, but, unlike `Cell<?>`, potentially unsafe operations such as invocation of `set` are (statically) allowed.

Our compatibility relation allows the opposite direction of flow, too—that is, an expression whose type involves dyn as a type argument can be assigned to a variable of a dyn-free type as in the following code:

```
Cell<Rectangle> c2 = c1;
Cell<Polygon> c3 = c1;
```

Just as an assignment of an expression of type dyn to a concrete type, the run-time system will check whether these are valid assignments: in this case, only the first assignment will succeed.

In summary, our compatibility relation allows dyn in a type expression to be replaced with a concrete type and vice versa. As we will see in the next section, however, its formal definition is more subtle than might have appeared when we take inheritance-based, nominal subtyping into account. Due to nominal subtyping, we take an approach different from the previous work.

2.2 Run-time Checks

In order to ensure that statically typed code (or, more precisely, code that would be well typed in the standard type system) will not go wrong, errors due to dynamically typed code have to be captured at the "border" between the two worlds. For this purpose, we introduce an intermediate language, which has explicit constructs for run-time checks; the semantics of the surface language, which we describe above, will be given in terms of the translation to the intermediate language.

Although there is no direct semantics for the surface language, a program in the surface language can be mostly directly understandable because the translation only inserts run-time checks and preserves the structure of a program. Moreover, run-time checks are inserted only where dynamic types are involved. So, as far as statically typed code is concerned, the translation is the identity map, without inserting any run-time checks. Then, in order to show that statically typed code never goes wrong, it suffices to show that all run-time failures are due to those explicit checks.

We will give an overview of constructs for run-time checks and the translation below. Table 1 shows constructs for run-time checks and their intuitive meanings. In what follows, we write $\mathtt{e} \rightsquigarrow \mathtt{e}'$ to mean that a surface language expression (or statement) e is translated to $\mathtt{e}'$.

First, when an expression of a type that involves dyn is passed to where a type without dyn is expected, a cast $(\!|\ |\!)$ is inserted:

```
Cell<Rectangle> c2; Cell<Polygon> c3;
c2 = c1;
  ⇝ c2 = (|Cell<Rectangle>|)c1;
c3 = c1;
  ⇝ c3 = (|Cell<Polygon>|)c1;
```

We use different parentheses $(\!|\ |\!)$ to denote casts because the semantics is slightly different from Java's. Note that the first cast above has to check that the run-time value of c1 is an instance of Cell<Rectangle> and not that of Cell<Integer>. So, this cast requires run-time type argument information. There are other differences, which we discuss later, as well.

$(\!\|\mathtt{T}\|\!)\mathtt{e}$	checks if the run-time type of the value of e is compatible with T, and then returns the value.
get(e,f)	checks if the value of e has field f, and then reduces to the field value.
$\mathtt{e.m[}\overline{\mathtt{T}}\mathtt{\|C<}\overline{\mathtt{X}}\mathtt{>](}\overline{\mathtt{e}}\mathtt{)}$	checks if the types of arguments $\overline{\mathtt{e}}$ are correct (using the static type information $\overline{\mathtt{T}}$, $\mathtt{C<}\overline{\mathtt{X}}\mathtt{>}$), and then invoke method m on receiver e.
$\mathtt{invoke(e,m,}\overline{\mathtt{e}}\mathtt{)}$	checks if the value of receiver e has method m and the types of arguments $\overline{\mathtt{e}}$ are correct, and then invoke the method on the receiver.

Table 1. Constructs for run-time checks.

A member access on dyn will be translated to get or invoke, which checks the existence of the member at run-time.

```
Cell<dyn> c1;
c1.x.radius;
  ⇝ get(c1.x, radius);
c1.x.contains(1, 1);
  ⇝ invoke(c1.x, contains, 1, 1);
```

When c1.x has method contains, invoke above also checks whether it can take two integers.

As we have already discussed, the invocation of set on type Cell<dyn> will have to check whether the run-time type of the argument is appropriate for the run-time type argument to the receiver's class, even though the existence of method set is statically guaranteed. For such cases, we use method invocation of the form $\mathtt{e}_0\mathtt{.m[}\mathtt{T}_1,\ldots,\mathtt{T}_n\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>](}\mathtt{e}_1,\ldots,\mathtt{e}_n\mathtt{)}$. For example, we have the following translation.

```
c1.set( new Polygon(...) );
  ⇝ c1.set[X|Cell<X>]( new Polygon(...) );
```

The annotations X and Cell<X> record a parameter type and a receiver's static type *before type parameters are instantiated* and are used to check the argument. It works as follows: When c1 evaluates to a value new Cell<T>(...) for some type T, the actual receiver type Cell<T> is matched against Cell<X> and X is bound to T. Then, the actual argument's type (here Polygon) is checked against T, which is obtained by replacing X with T in the recorded parameter type. So, this method invocation succeeds when c1's value is an object of Cell<Polygon> (or Cell<T> where T is a supertype of Polygon).

2.3 Ensuring "statically typed parts cannot go wrong"

One of our goals of the gradual type system is to ensure that "statically typed parts in a program never go wrong", in particular, class definitions that pass the standard type checker should not go wrong. Another desirable property of the system is modularity of type checking, that is, determining whether the given part of the program is statically typed or not should be done by looking at no more than a single class definition and type information that it depends on. We also aim at implementation by erasure translation [8].[1] Actually, modular checking and erasure translation make it trickier to ensure the safety of "statically typed code".

First of all, even if a class definition contains no occurrence of `dyn`, it should not be considered statically typed because subexpressions may be given type `dyn`. So, a sensible definition of a statically typed class definition is something like "a class definition is statically typed if there is no occurrence of `dyn` and every subexpression is given a '`dyn`-free' type." In fact, as we will see later, in our translation, a method invocation requires no run-time check if the receiver and actual argument types are all `dyn`-free. Then, a class definition that passes the standard type checking will translates to itself, without run-time checks.

However, the problem is more subtle than it might have appeared, due to the presence of type variables. In general, type variables should not be considered `dyn`-free simply because type variables can be instantiated with `dyn`. In fact, the following classes, which are typed under the standard type system of generics

```
class StrCell extends Cell<String> {
  void set(String x){ ... x.length() ... }
}
class Foo<Y> {
  void bar(Cell<Y> c, Y x) {
    c.set(x);
  }
}
```

will raise a run-time error when combined with the following code:

```
new Foo<dyn>().bar(new StrCell(...), new Object());
```

The last expression passes a `StrCell` and an `Object` to `Foo<dyn>.bar()`, which expects a `Cell<dyn>` and `dyn`, and this is allowed due to the extended compatibility relation we have already mentioned. In `Foo<Y>.bar()`, an object `x` is passed to `Cell<Y>.set()`. However, in this case, the receiver is a `StrCell` and `StrCell.set()` will be called with an argument `new Object()`, which does not have `length()`!

To avoid this problem, we separate type variables into two kinds: one can only be replaced with `dyn`-free types, and the other can be replaced with dynamic types. These kinds are indicated in the class definition, for example, `class Foo<Y`$^{\Diamond}$`>...` for the former and `class Foo<Y`$^{\blacklozenge}$`>...` for the latter. If `Foo` is defined as `class Foo<Y`$^{\Diamond}$`>...`, then no run-time check is inserted but the problematic expression above is rejected at compile time. Otherwise, if `Foo` is defined as `class Foo<Y`$^{\blacklozenge}$`>...`, then the invocation of `set` will check whether the actual argument types are valid for the formal (by using `e.m[`$\overline{\texttt{T}}$`|C<`$\overline{\texttt{X}}$`>](`$\overline{\texttt{e}}$`)`).

Although, in principle, a programmer can choose the kind for each type variable declaration in a single class definition, we do not expect that a programmer wants to do it. A practical design would be that a compiler option will decide the kind of all the type variables in a compiled file at once. In the beginning of development, the programmer may compile most generic classes with kind $\blacklozenge$, and then switches some classes to $\Diamond$ gradually as the development progresses—such a switch would enforce their client code to remove the use of dynamic types. In the rest of the paper, we omit kinds of type variables when they do not make significant difference.

2.4 Bounded Dynamic Types

Another problem occurs when a type variable is given an upper bound. To illustrate the problem, consider the following class:

```
class ShapeCell<X♦ extends Shape> {
  X x;  ShapeCell(X x){ this.x=x; }
  void set(X x){ this.x=x; }
  boolean contains(double px, double py){
    return this.x.contains(px, py);
  }
}
```

Class `ShapeCell`, which is similar to class `Cell` above, specifies `Shape` as `X`'s upper bound. Note that `ShapeCell` does not contain `dyn` anywhere and the whole class definition will be well typed in the standard type system of generics.

Now consider type `ShapeCell<dyn>`. The question here is what we can set to `x`. One choice would be to allow any object to be set to `x`, as we did for `Cell<dyn>`:

```
ShapeCell<dyn> sc = ...;
sc.set( new Object() );
```

However, this choice would not be compatible with the implementation by erasure, which translates the type of field `x` to be `Shape`, the upper bound of `X`. In a language without erasure semantics, for example in C♯, this choice would not conflict with the implementation, but, as long as homogeneous translation [24] is used, we need to treat type variable `X` with kind $\blacklozenge$ as `dyn`. This leads to a major performance disadvantage since operations on expressions of type `X` need to be augmented with run-time checks that require membership tests: for example in the case above, the invocation of `contains` on `this.x` need to be replaced with an expensive run-time check by `invoke`, which checks the presence of method `contains` and (if exists) whether the types of for-

[1] Our compilation scheme actually requires support for run-time type arguments. However, method signatures are subject to erasure.

mal and actual arguments match. Thus, our choice here is to keep compatibility with the implementation by erasure and to avoid performance penalty as much as possible: in other words, we reject the code above statically.

We introduce *bounded dynamic types*, written `dyn<T>` (where `T` stands for a parametric type). A type parameter with an upper bound `T` can be instantiated by a bounded dynamic type `dyn<T'>` when `T'` is a subtype of `T`. Thus, `ShapeCell<dyn<Shape>>` is a well-formed type, whereas `ShapeCell<dyn<Object>>` is not.

We define a bounded dynamic type `dyn<T>` to be compatible only with subtypes of `T`. So, the following code will be ill typed and rejected by the type checker.

```
ShapeCell<dyn<Shape>> sc = ...;
sc.set( new Object() );
// Object is not compatible with dyn<Shape>!
```

A bounded dynamic type has both static and dynamic typing natures. While it still allows potentially unsafe operations to be performed, it enforces static typing as far as members defined in the bound are concerned. So, the first two lines in the following code are still accepted (and checked at run time) but not the third[2] and fourth.

```
sc.x.anyField;
sc.x.anyMethod(...);
sc.x.contains(); // two arguments are expected!
Shape s = sc.x.contains(3, 4); // returns boolean!
```

In a real language, we do not expect programmers to write those upper bounds explicitly. Rather, when `dyn` is used as a type argument, the compiler can recover its upper bound automatically by assigning the upper bounds of the corresponding type parameters in the generic class definition.[3] For other uses of `dyn`, they can be regarded as `dyn<Object>`; in fact, we use `dyn` as an abbreviation of `dyn<Object>`, throughout the paper.

2.5 Two Compatibility Relations

As we have already mentioned, the type system of the surface language uses the compatibility relation, denoted by $\lesssim$, to check argument passing and assignments. This relation has both co- and contra-variant flavors when `dyn` is considered a top type: for example, both `Cell<Shape>` $\lesssim$ `Cell<dyn>` and `Cell<dyn>` $\lesssim$ `Cell<Shape>` hold and so both

```
Cell<Shape> c1 = ...; Cell<dyn> c2 = c1;
```

and

```
Cell<dyn> c1 = ...;  Cell<Shape> c2 = c1;
```

are accepted (statically). The reason to allow contravariance (the latter kind of compatibility) is simply because it *sometimes* runs safely. For example, when `c1` happens to be an object `new Cell<Shape>(...)`, the latter code fragment is just fine.

However, we should not use this compatibility relation for casts. For example, consider the following (surface language) code:

```
Cell<dyn> c1 = new Cell<dyn>(new Polygon(...));
Cell<Rectangle> c2 = c1;      // accepted thanks
                              // to contravariance
c2.x.methodOnlyInRectangle(); // accepted since
                              // c2.x is Rectangle
```

On the second line, a run-time check $(\!|$`Cell<Rectangle>`$|\!)$`c1` is performed. Since the run-time type of `c1` is `Cell<dyn>`, if $(\!|\,|\!)$ used the compatibility relation, the cast would succeed, resulting in the unexpected method-not-found error! (Notice that the invocation of `methodOnlyInRectangle` should involve no checks because the receiver's static type does not contain `dyn`.)

Thus, we use another relation $\precsim$ called *run-time compatibility* for run-time checks. This relation is a subrelation of $\lesssim$ and disallows contravariance: for example, `Cell<dyn>` $\precsim$ `Cell<Rectangle>` does not hold. However, it still allows covariance (such as `Cell<Rectangle>` $\precsim$ `Cell<dyn>`), so it is more permissive than subtyping. (It is not completely safe—that is why we still need argument checks by $\mathtt{e}_0\mathtt{.m[T}_1,\ldots,\mathtt{T}_n\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>](e}_1,\ldots,\mathtt{e}_n\mathtt{)}$.)

Having these discussions in mind, we formalize the core of the surface and intermediate languages in the following sections.

3. Featherweight GJ with Dynamic Types

In this section, we formalize the surface language FGJ$^{\mathtt{dyn}}$, an extension of FGJ with dynamic types to model a type system of gradually typed generics. For simplicity, we omit some features found in FGJ: F-bounded polymorphism, polymorphic methods, and typecasts, which would be easy to add. We focus on the type system in this section and leave the definition of the intermediate language called FGJ$^{(\!|\,|\!)}$ and translation from FGJ$^{\mathtt{dyn}}$ to FGJ$^{(\!|\,|\!)}$. For those who are familiar with FGJ, we use gray boxes to show differences from FGJ.

3.1 Syntax and Lookup Functions

The abstract syntax of FGJ$^{\mathtt{dyn}}$ classes, constructors and method declarations, and expressions are defined as follows:

Definition 1 (Syntax of FGJ$^{\mathtt{dyn}}$)**.**

κ, ι ::= ♦ | ◊
S, T, U, V ::= X | N | `dyn<N>`
N, P, Q ::= `C<`$\overline{\mathtt{T}}$`>`
e ::= `x` | `new N(`$\overline{\mathtt{e}}$`)` | `e.f` | `e.m(`$\overline{\mathtt{e}}$`)`
L ::= `class C<`$\overline{\mathtt{X}}^{\kappa}$ $\triangleleft$ $\overline{\mathtt{N}}$`>` $\triangleleft$ N `{`$\overline{\mathtt{T}}$ $\overline{\mathtt{f}}$`; K` $\overline{\mathtt{M}}$`}`
K ::= `C(`$\overline{\mathtt{T}}$ $\overline{\mathtt{f}}$`){super(`$\overline{\mathtt{f}}$`); this.`$\overline{\mathtt{f}}$`=`$\overline{\mathtt{f}}$`;}`
M ::= `T m(`$\overline{\mathtt{T}}$ $\overline{\mathtt{x}}$`){ return e; }`

[2] It could be allowed in the presence of overloading.

[3] For F-bounded type variables, such automatic recovery is difficult. We would have to have programmers write upper bounds explicitly.

The metavariables A, B, C, D and E range over class names; W, X, Y and Z range over type variables; N, P and Q range over class types (dyn<N> is not a class type); S, T, U and V range over types; κ and ι range over kinds of type variables; f and g range over field names; m ranges over method names; x ranges over variables; d and e range over expressions; L ranges over class declarations; K ranges over constructor declarations; and M ranges over method declarations. We assume that the set of variables includes the special variable this.

We write $\overline{\mathtt{f}}$ as shorthand for a possibly empty sequence $\mathtt{f}_1,\mathtt{f}_2,\ldots,\mathtt{f}_n$ (and similarly for $\overline{\mathtt{C}}$, $\overline{\mathtt{X}}$, $\overline{\mathtt{N}}$, $\overline{\mathtt{T}}$, $\overline{\mathtt{x}}$, $\overline{\mathtt{e}}$, etc.) and write $\overline{\mathtt{M}}$ as shorthand for $\mathtt{M}_1 \ldots \mathtt{M}_n$ (with no commas). The length of a sequence $\overline{\mathtt{f}}$ is written $\#(\overline{\mathtt{f}})$. We write $\mathtt{f} \in \overline{\mathtt{f}}$ when f equals to $\mathtt{f}_i$ where $1 \leq i \leq \#(\overline{\mathtt{f}})$, and write $\mathtt{f} \notin \overline{\mathtt{f}}$ otherwise. We also abbreviate various forms of sequences of pairs in the obvious way, writing "$\overline{\mathtt{T}}\ \overline{\mathtt{f}}$" as shorthand for "$\mathtt{T}_1\ \mathtt{f}_1, \ldots, \mathtt{T}_n\ \mathtt{f}_n$" where $\#(\overline{\mathtt{T}}) = \#(\overline{\mathtt{f}})$, and similarly "$\overline{\mathtt{T}}\ \overline{\mathtt{f}};$" for the sequence of declarations "$\mathtt{T}_1\ \mathtt{f}_1; \cdots \mathtt{T}_n \mathtt{f}_n;$", "$\mathtt{this.}\overline{\mathtt{f}}\mathtt{=}\overline{\mathtt{f}}\mathtt{;}$" for "$\mathtt{this.f}_1\mathtt{=f}_1\mathtt{;} \cdots \mathtt{this.f}_n\mathtt{=f}_n\mathtt{;}$", and "$\overline{\mathtt{X}^{\overline{\kappa}} \lhd \mathtt{N}}$" for "$\mathtt{X}_1^{\kappa_1} \lhd \mathtt{N}_1, \ldots, \mathtt{X}_n^{\kappa_n} \lhd \mathtt{N}_n$" . We write the empty sequence as $\bullet$ and denote concatenation of sequences using a comma. Sequences are assumed to contain no duplicate names. We abbreviate the keyword extends to the symbol $\lhd$.

dyn<N> is a type of dynamically typed expressions. Since it is not a class type, dyn<N> can neither be used to instantiate an object (by new expressions) nor be used as a bound of a type variable. We always write the bound N in the formal language when N is not Object, but the real programming language should allow bounds to be omitted from the source code. As we discussed in the previous section, it is easy to recover the bounds for most cases. Similarly, it is reasonable to treat C without the arguments as C<dyn, ..., dyn>.

A program in FGJ$^{\mathtt{dyn}}$ is a pair $(CT, \mathtt{e})$ of a class table, which is a finite mapping from class names C to class declarations L, and a closed expression corresponding to the body of the main method. We assume that CT satisfies some sanity conditions: (1) $CT(\mathtt{C}) = \mathtt{class\ C<}\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \ldots \mathtt{\{\ldots\}}$ for every $\mathtt{C} \in \mathit{dom}(CT)$; (2) $\mathtt{Object} \notin \mathit{dom}(CT)$; (3) for every class name C (except Object) appearing anywhere in CT, we have $\mathtt{C} \in \mathit{dom}(CT)$; and (4) there are no cycles in the transitive closure of $\lhd$ (as a relation between class names). In what follows, we fix a class table.

As in FGJ, we use functions *fields* and *mtype* to look up field definitions and method types in a given class table. We also use a predicate *nomethod* to state non-existence of a method. We omit their straightforward definitions (see appendix or Igarashi, Pierce, and Wadler [16] for the definitions of *fields* and *mtype*); their functionalities are summarized in Table 2.

Some other auxiliary functions are defined in Figure 1. We use a function *bound* to compute the upper bound of a type in a bound environment Δ, which is a finite sequence of triples of a type variable, its kind and its bound, where type variables are pairwise distinct.[4] When *bound* is used with a class type, it returns the given type itself. When *bound* is used with a dynamic type, it returns the bound of the dynamic type. We have a function *kind* to look up kinds of type variables. We use a predicate *dynfree* to state that a type is dynamic-free, i.e., it contains no dynamic type.

$\mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$	collects fields in class N and its super type.
$\mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{T}} \to \mathtt{T}$	looks up the type of method m in class N or its super type.
$\mathit{nomethod}(\mathtt{m}, \mathtt{N})$	holds when there is no method m in class N or its super type.

Table 2. Definition of FGJ$^{\mathtt{dyn}}$: Auxiliary functions and predicates.

3.2 Subtyping and Compatibility

Now we define subtyping and compatibility relations. As we have mentioned, there are two compatibility relations (written $\nprec$ for run-time compatibility and $\lesssim$ for static compatibility). We write $\Delta \vdash \mathtt{S} <: \mathtt{T}$ to mean S is a subtype of T under bound environment Δ. Similarly for $\Delta \vdash \mathtt{S} \nprec \mathtt{T}$ and $\Delta \vdash \mathtt{S} \lesssim \mathtt{T}$. We abbreviate a sequence of judgments $\Delta \vdash \mathtt{S}_1 <: \mathtt{T}_1, \ldots, \Delta \vdash \mathtt{S}_n <: \mathtt{T}_n$ to $\Delta \vdash \overline{\mathtt{S}} <: \overline{\mathtt{T}}$ (and similarly for $\nprec$ and $\lesssim$).

Definition 2 (FGJ$^{\mathtt{dyn}}$ subtyping and compatibility)**.** The subtyping and compatibility judgments $\Delta \vdash \mathtt{S} <: \mathtt{T}$ and $\Delta \vdash \mathtt{S} \nprec \mathtt{T}$ and $\Delta \vdash \mathtt{S} \lesssim \mathtt{T}$ are defined by the rules in Figure 2.

The subtype relation $<:$ is mostly the same as that of FGJ. The first two rules mean that it is reflexive and transitive; the third rule that a type variable is a subtype of its bound; the fourth rule is about inheritance-based subtyping—any instance of a $\lhd$ clause gives subtyping. The last rule says a bounded dynamic type dyn<N> is a subtype of its bound N. In fact, dyn<N> and N denote the same set of instances—instances of N and its subtypes and dyn<N> allows *more* operations (which are potentially unsafe, though) to be performed than N. So, dyn<N> <: N indeed agrees with the substitution principle [22].

The compatibility relations are defined with the help of an auxiliary relation $\prec$. Intuitively, $\mathtt{S} \prec \mathtt{T}$ means that T is obtained by replacing some class types in S with dynamic types (with an appropriate bound). For example, Rectangle $\prec$ dyn<Object> and Cell<Rectangle> $\prec$ Cell<dyn<Object>> hold (under any bound environment). So, $\prec$ represents a form of covariance. Then, the compatibility relations are defined as compositions of $\prec$ and $<:$—the former as $(<:; \prec)$ ($\cdot;\cdot$ is a composition of two relations)

[4] So, Δ can be considered a finite mapping.

Bound environment

$$\frac{\Delta(\mathtt{X}) = (\kappa, \mathtt{N})}{bound_\Delta(\mathtt{X}) = \mathtt{N}} \qquad bound_\Delta(\mathtt{N}) = \mathtt{N} \qquad bound_\Delta(\mathtt{dyn<N>}) = \mathtt{N} \qquad \frac{\Delta(\mathtt{X}) = (\kappa, \mathtt{N})}{kind_\Delta(\mathtt{X}) = \kappa}$$

Dynamic-free types

$$\frac{kind_\Delta(\mathtt{X}) = \Diamond}{dynfree_\Delta(\mathtt{X})} \qquad \frac{dynfree_\Delta(\overline{\mathtt{T}})}{dynfree_\Delta(\mathtt{C<}\overline{\mathtt{T}}\mathtt{>})}$$

Figure 1. Definition of FGJ$^{\mathtt{dyn}}$: Auxiliary functions and predicates.

Subtyping

$$\Delta \vdash \mathtt{T} <: \mathtt{T} \qquad \frac{\Delta \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta \vdash \mathtt{U} <: \mathtt{T}}{\Delta \vdash \mathtt{S} <: \mathtt{T}} \qquad \Delta \vdash \mathtt{X} <: bound_\Delta(\mathtt{X})$$

$$\frac{\mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \mathtt{\{...\}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{T}}\mathtt{>} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}} \qquad \Delta \vdash \mathtt{dyn<N>} <: \mathtt{N}$$

Compatibility

$$\Delta \vdash \mathtt{T} \prec \mathtt{T} \qquad \frac{\Delta \vdash \mathtt{S} \prec \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{T}}{\Delta \vdash \mathtt{S} \prec \mathtt{T}} \qquad \frac{\Delta \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{T}}{\Delta \vdash \mathtt{S} \nprec \mathtt{T}}$$

$$\frac{\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{T}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{S}}\mathtt{>} \prec \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}} \qquad \frac{\Delta \vdash \mathtt{T} <: \mathtt{S} \qquad \Delta \vdash \mathtt{S} \prec \mathtt{N}}{\Delta \vdash \mathtt{T} \prec \mathtt{dyn<N>}} \qquad \frac{\Delta \vdash \mathtt{U} \prec \mathtt{S} \qquad \Delta \vdash \mathtt{U} \nprec \mathtt{T}}{\Delta \vdash \mathtt{S} \lesssim \mathtt{T}}$$

Figure 2. Definition of FGJ$^{\mathtt{dyn}}$: Subtyping and compatibility.

and the latter as $(\prec^{-1}; <:; \prec)$, where $\prec^{-1}$ is the inverse of $\prec$ and represents a form of contravariance. As a result, two types are statically compatible if replacing dynamic types with class types yields two types in the subtyping relation.

For example,

$$\Delta \vdash \mathtt{Cell<dyn>} \lesssim \mathtt{Cell<Rectangle>}$$

can be derived since `Cell<dyn>` $\prec^{-1}$ `Cell<Rectangle>`. Also,

$$\Delta \vdash \mathtt{Cell<dyn<Shape>>} \lesssim \mathtt{Cell<dyn>}$$

since `Cell<dyn<Shape>>` $\prec^{-1}$ `Cell<Shape>` and `Cell<Shape>` $\prec$ `Cell<dyn>`. (Note that `Cell<dyn<Shape>>` $<:$ `Cell<dyn>` does *not* hold.)

`dyn<Object>` can be considered either a top type or a bottom type, i.e., $\Delta \vdash \mathtt{T} \lesssim \mathtt{dyn<Object>}$ and $\Delta \vdash \mathtt{dyn<Object>} \lesssim \mathtt{T}$ are satisfied for any `T`, and the relation $\lesssim$ is not transitive because otherwise $\Delta \vdash \mathtt{S} \lesssim \mathtt{T}$ would be implied for any `S`,`T` (as mentioned in [25, 26]). `dyn<N>` can also be considered as a top/bottom type for subtypes of `N`.

3.3 Type Well-formedness and Typing

We, then, define well-formed types and typing.

Definition 3 (FGJ$^{\mathtt{dyn}}$ type well-formedness)**.** The type well-formedness judgment $\Delta \vdash \mathtt{T}$ ok, read as "in bound environment Δ, type `T` is well formed," is defined by the rules in Figure 3.

The last rule means that a bounded dynamic type is well formed if its upper bound is well formed. The third rule means that a class type is well formed if well formed type arguments that satisfy the corresponding type parameters' upper bounds. Note that we use $\nprec$ rather than $<:$ or $\lesssim$. First, $<:$ cannot be used because we want to use dynamic types as type arguments. For example, `Cell<dyn>` is well formed because `dyn` $\nprec$ `Object`. $\lesssim$ should not be used, either, because we want to reject a type like `ShapeCell<dyn>`. (Note that `dyn` $\lesssim$ `Shape`.) The third rule also requires that type arguments must be dynamic-free if the kind of the corresponding type variable is $\Diamond$.

Now we are ready to define typing. We use Γ as a type environment, which is a finite mapping from variables to types, written $\overline{\mathtt{x}}\mathtt{:}\overline{\mathtt{C}}$.

Definition 4 (FGJ$^{\mathtt{dyn}}$ typing)**.** The type judgments $\Delta; \Gamma \vdash_{\mathrm{G}} \mathtt{e} : \mathtt{T}$, read as "in environment Δ and Γ, expression `e` has type `T`," and `M OK IN` $\mathtt{C<}\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\mathtt{>}$, read as "method `M` is well-formed in class $\mathtt{C<}\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\mathtt{>}$" and `L OK`, read as "class `L` is well-formed" are defined as in Figure 4.

$$\Delta \vdash \texttt{Object}\ \text{ok} \qquad \frac{\texttt{X} \in \mathit{dom}(\Delta)}{\Delta \vdash \texttt{X}\ \text{ok}} \qquad \frac{\begin{array}{c}\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{>} \lhd \texttt{N}\ \texttt{\{...\}} \qquad \Delta \vdash \overline{\texttt{T}}\ \text{ok} \\ \forall \kappa_i \in \overline{\kappa}.(\ \kappa_i = \Diamond \text{ implies } \mathit{dynfree}_\Delta(\texttt{T}_i)\ ) \\ \forall \texttt{T}_i \in \overline{\texttt{T}}.(\ \Delta \vdash \texttt{T}_i \prec [\texttt{T}_1/\texttt{X}_1, \ldots, \texttt{T}_{i-1}/\texttt{X}_{i-1}]\texttt{N}_i\ )\end{array}}{\Delta \vdash \texttt{C<}\overline{\texttt{T}}\texttt{>}\ \text{ok}} \qquad \frac{\Delta \vdash \texttt{N}\ \text{ok}}{\Delta \vdash \texttt{dyn<N>}\ \text{ok}}$$

Figure 3. Definition of FGJ$^{\texttt{dyn}}$: Type well-formedness.

Expression typing

$$\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{x} : \Gamma(\texttt{x})\ (\textsc{Tg-Var}) \qquad \frac{\Delta \vdash_{\mathrm{G}} \texttt{N}\ \text{ok} \qquad \mathit{fields}(\texttt{N}) = \overline{\texttt{T}}\ \overline{\texttt{f}} \qquad \Delta; \Gamma \vdash_{\mathrm{G}} \overline{\texttt{e}} : \overline{\texttt{U}} \qquad \Delta \vdash \overline{\texttt{U}} \lesssim \overline{\texttt{T}}}{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{new N}(\overline{\texttt{e}}) : \texttt{N}}\ (\textsc{Tg-New})$$

$$\frac{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{e}_0 : \texttt{T}_0 \qquad \mathit{fields}(\mathit{bound}_\Delta(\texttt{T}_0)) = \overline{\texttt{T}}\ \overline{\texttt{f}}}{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{e}_0.\texttt{f}_i : \texttt{T}_i}\ (\textsc{Tg-Field1}) \qquad \frac{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{e}_0 : \texttt{dyn<N>} \qquad \texttt{f} \notin \overline{\texttt{f}} \qquad \mathit{fields}(\texttt{N}) = \overline{\texttt{T}}\ \overline{\texttt{f}}}{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{e}_0.\texttt{f} : \texttt{dyn<Object>}}\ (\textsc{Tg-Field2})$$

$$\frac{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{e}_0 : \texttt{T}_0 \qquad \Delta; \Gamma \vdash_{\mathrm{G}} \overline{\texttt{e}} : \overline{\texttt{V}} \qquad \mathit{mtype}(\texttt{m}, \mathit{bound}_\Delta(\texttt{T}_0)) = \overline{\texttt{S}} \to \texttt{S} \qquad \Delta \vdash \overline{\texttt{V}} \lesssim \overline{\texttt{S}}}{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{e}_0.\texttt{m}(\overline{\texttt{e}}) : \texttt{S}}\ (\textsc{Tg-Invk1})$$

$$\frac{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{e}_0 : \texttt{dyn<N>} \qquad \Delta; \Gamma \vdash_{\mathrm{G}} \overline{\texttt{e}} : \overline{\texttt{V}} \qquad \mathit{nomethod}(\texttt{m}, \texttt{N})}{\Delta; \Gamma \vdash_{\mathrm{G}} \texttt{e}_0.\texttt{m}(\overline{\texttt{e}}) : \texttt{dyn<Object>}}\ (\textsc{Tg-Invk2})$$

Method typing

$$\frac{\mathit{mtype}(\texttt{m}, \texttt{N}) = \overline{\texttt{U}} \to \texttt{U} \text{ implies } \overline{\texttt{T}} = \overline{\texttt{U}} \text{ and } \Delta \vdash \texttt{T} \prec \texttt{U}}{\mathit{override}_\Delta(\texttt{m}, \texttt{N}, \overline{\texttt{T}} \to \texttt{T})}$$

$$\frac{\begin{array}{c}\Delta = \overline{\texttt{X}}^{\overline{\kappa}} <: \overline{\texttt{N}} \qquad \Delta \vdash \overline{\texttt{T}}, \texttt{T}\ \text{ok} \qquad \Delta; \overline{\texttt{x}} : \overline{\texttt{T}}, \texttt{this} : \texttt{C<}\overline{\texttt{X}}\texttt{>} \vdash_{\mathrm{G}} \texttt{e}_0 : \texttt{S} \\ \Delta \vdash \texttt{S} \lesssim \texttt{T} \qquad \texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{>} \lhd \texttt{N}\ \texttt{\{...\}} \qquad \mathit{override}_\Delta(\texttt{m}, \texttt{N}, \overline{\texttt{T}} \to \texttt{T})\end{array}}{\texttt{T m(}\overline{\texttt{T}}\ \overline{\texttt{x}}\texttt{)\{ return e}_0\texttt{; \}}\ \texttt{OK IN C<}\overline{\texttt{X}} \lhd \overline{\texttt{N}}\texttt{>}}\ (\textsc{Tg-Method})$$

Class typing

$$\frac{\begin{array}{c}\forall \texttt{N}_i \in \overline{\texttt{N}}.(\texttt{X}_1^{\kappa_1} <: \texttt{N}_1, \ldots, \texttt{X}_{i-1}^{\kappa_{i-1}} <: \texttt{N}_{i-1} \vdash \texttt{N}_i\ \text{ok}) \qquad \overline{\texttt{X}}^{\overline{\kappa}} <: \overline{\texttt{N}} \vdash \texttt{N}, \overline{\texttt{T}}\ \text{ok} \\ \mathit{fields}(\texttt{N}) = \overline{\texttt{U}}\ \overline{\texttt{g}} \qquad \overline{\texttt{M}}\ \texttt{OK IN C<}\overline{\texttt{X}} \lhd \overline{\texttt{N}}\texttt{>} \qquad \texttt{K} = \texttt{C(}\overline{\texttt{U}}\ \overline{\texttt{g}}\texttt{, }\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{)\{super(}\overline{\texttt{g}}\texttt{); this.}\overline{\texttt{f}}\texttt{=}\overline{\texttt{f}}\texttt{;\}}\end{array}}{\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{> } \lhd \texttt{ N }\{\overline{\texttt{T}}\ \overline{\texttt{f}}\texttt{; K }\ \overline{\texttt{M}}\texttt{\} OK}}\ (\textsc{Tg-Class})$$

Figure 4. Definition of FGJ$^{\texttt{dyn}}$: Typing.

Most of the rules are straightforward adaptation of those in FGJ [16], except that the relation $\lesssim$ is substituted for $<:$. TG-FIELD2 and TG-INVK2 are additional rules, used when the receiver type is a bounded dynamic type. Note that these rules are applied only when it is not known whether the receiver has a field or method to be accessed (the premises $\texttt{f} \notin \overline{\texttt{f}}$ in TG-FIELD2 and $\mathit{nomethod}(\texttt{m}, \texttt{N})$ in TG-INVK2). This gives `dyn<N>` a characteristic of ordinary class types.

The predicate *override*, used in method typing, is to check if a method `m` in class `N` can be overridden by a method of type $\overline{\texttt{T}} \to \texttt{T}$. Parameter types must be the same between the overriding and overridden methods and the return type of the overriding method must be run-time compatible with that of the overridden method. We cannot use $\lesssim$ here: if $\lesssim$ were used, it would be possible to override a method that returns `T` by one that returns `dyn<Object>` in a subclass, and then to override it by another that returns `S` for *any* `S`. As a result, invoking a method, whose static return type is `T`, might actually invoke the third method that returns `S`, which can be very different from `T`, due to late binding!

We write $\vdash_{\mathrm{G}} (CT, \mathtt{e}) : \mathtt{T}$ to mean the program is well formed, i.e, if all the classes in CT are well formed and e is well typed under the empty environments.

We have not completed to develop a type checking algorithm. In fact, it is not even clear that the compatibility relation $\lesssim$ is decidable for the same reason as variance-based subtyping [20].

Conservative Typing over FGJ. Even at this point, we can show an interesting property that typing in FGJ$^{\mathtt{dyn}}$ is a conservative extension of that in FGJ. Namely, as far as a class table written in the FGJ syntax is concerned, it is well typed under the FGJ rules if and only if it is well typed under the FGJ$^{\mathtt{dyn}}$ rules. The following lemma is a key to the conservative extension property (Theorem 6).

Lemma 5.

- If $\Delta \vdash \mathtt{S} \prec \mathtt{T}$ and $\mathit{dynfree}_{\Delta}(\mathtt{T})$, then $\mathtt{S} = \mathtt{T}$.
- If $\Delta \vdash \mathtt{S} \nprec \mathtt{T}$ and $\mathit{dynfree}_{\Delta}(\mathtt{T})$, then $\Delta \vdash \mathtt{S} <: \mathtt{T}$.
- If $\Delta \vdash \mathtt{S} \lesssim \mathtt{T}$ and $\mathit{dynfree}_{\Delta}(\mathtt{S})$, then $\Delta \vdash \mathtt{S} \nprec \mathtt{T}$.
- If $\Delta \vdash \mathtt{S} \lesssim \mathtt{T}$ and $\mathit{dynfree}_{\Delta}(\mathtt{S})$ and $\mathit{dynfree}_{\Delta}(\mathtt{T})$, then $\Delta \vdash \mathtt{S} <: \mathtt{T}$.

We write $\vdash_{\mathrm{FGJ}} (CT, \mathtt{e}) : \mathtt{T}$ if the program, which does not contain `dyn<N>`, is well formed under the FGJ rules.

Theorem 6 (FGJ$^{\mathtt{dyn}}$ Typing is Conservative over FGJ Typing)**.** If $\overline{\kappa} = \overline{\Diamond}$ for every `class C<`$\overline{\mathtt{X}}^{\overline{\kappa}}$`> ◁ `$\overline{\mathtt{N}}$`> ◁ N {...}` in CT and none of `dyn<P>` appears in $(CT, \mathtt{e})$, then $\vdash_{\mathrm{FGJ}} (CT, \mathtt{e}) : \mathtt{T} \iff \vdash_{\mathrm{G}} (CT, \mathtt{e}) : \mathtt{T}$.

4. From FGJ$^{\mathtt{dyn}}$ to FGJ$^{(\!(\ )\!)}$

In this section, we first define a formal model FGJ$^{(\!(\ )\!)}$ of the intermediate language, into which source programs are translated. FGJ$^{(\!(\ )\!)}$ has operational semantics, as well as a type system. After stating theorems about type safety of FGJ$^{(\!(\ )\!)}$, we present formal translation from FGJ$^{\mathtt{dyn}}$ to FGJ$^{(\!(\ )\!)}$ and state theorems that translation preserves typing. We have weak and strong versions of type safety: the weak version means that a well typed program can raise errors only at run-time checks and the strong version means that a well typed program without containing run-time checks never goes wrong in the usual sense.

4.1 The Target Language FGJ$^{(\!(\ )\!)}$

The syntax of FGJ$^{(\!(\ )\!)}$ extends that of FGJ$^{\mathtt{dyn}}$, by including special forms for run-time checks and run-time errors. We show only the grammar for expressions, values (used to define the semantics), and errors; the others are the same.

Definition 7 (Syntax of FGJ$^{(\!(\ )\!)}$)**.**

$$
\begin{array}{lcl}
\mathtt{e} & ::= & \mathtt{x} \mid \mathtt{new\ N(}\overline{\mathtt{e}}\mathtt{)} \mid \mathtt{e.f} \mid \mathtt{e.m(}\overline{\mathtt{e}}\mathtt{)} \\
& & \mid \mathtt{get(e,f)} \mid \mathtt{e.m[}\overline{\mathtt{T}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>](}\overline{\mathtt{e}}\mathtt{)} \\
& & \mid \mathtt{invoke(e,m,}\overline{\mathtt{e}}\mathtt{)} \mid (\!(\mathtt{N})\!)\,\mathtt{e} \\
& & \mid \mathtt{Error[}\mathcal{E}\mathtt{]} \\
\mathtt{v}, \mathtt{w} & ::= & \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{)} \\
\mathcal{E} & ::= & \mathtt{NoSuchField} \mid \mathtt{NoSuchMethod} \\
& & \mid \mathtt{IllegalArgument} \mid \mathtt{BadCast}
\end{array}
$$

We avoid repeating the definitions of the following functions, predicates, and relations since they are defined exactly the same way as in FGJ$^{\mathtt{dyn}}$.

Functions	bound
	kind
	fields
	mtype
Predicates	$\mathit{dynfree}$
	$\mathit{nomethod}$
	$\mathit{override}$
Relations	Subtyping $<:$
	Compatibility $\prec$, $\nprec$
Judgments	Type well-formedness

Some more auxiliary functions are listed in Figure 5. $\langle \mathtt{S} \Leftarrow \mathtt{T} \rangle_{\Delta} \mathtt{e}$ inserts a cast when source type T is not run-time compatible with S. This reduces unnecessary casts. $\mathit{tyargs}(\mathtt{N}, \mathtt{C})$ is used in the reduction rules described later to get type arguments from run-time types. We also use a function $\mathit{mbody}(\mathtt{m}, \mathtt{N})$, which returns a pair $\overline{\mathtt{x}}.\mathtt{e}$ of a sequence of formal parameters and a method body expression, if N has m. Its straightforward definition is omitted.

We show the main typing rules of FGJ$^{(\!(\ )\!)}$ in Figure 6. An important point to note is that we use only the run-time compatibility $\nprec$ and no $\lesssim$ appears in the typing rules because a use of $\lesssim$ compiles to a cast $(\!(\ )\!)$, which uses $\nprec$ for its run-time check. TR-INVK1 is the rule for a method invocation without run-time checks. The receiver type must be dyn-free. TR-INVK2 is the rule for a method invocation with run-time argument checking. `C<`$\overline{\mathtt{T}}$`>` can be considered an initial (i.e., compile-time) static type of receiver $\mathtt{e}_0$; `N` $\nprec$ `C<`$\overline{\mathtt{T}}$`>` is required since the receiver type may change as reduction proceeds. TR-INVK3 is the rule for a method invocation that checks whether the method m exists at run time. The receiver and arguments can be arbitrary typeable expressions and the type of the invocation is `dyn<Object>`. TR-CAST is the rule for casts; and TR-ERROR is the rule for errors.

We omit typing rules for object constructions, field accesses, methods and classes, since they are similar to those in FGJ$^{\mathtt{dyn}}$. We write $\vdash_{\mathrm{R}} (CT, \mathtt{e}) : \mathtt{T}$ to mean the FGJ$^{(\!(\ )\!)}$ program $(CT, \mathtt{e})$ is well formed.

We give main reduction rules in Figure 7. The evaluation order is, unlike FGJ, fixed to be left-to-right and call-by-value to deal with run-time errors more precisely. R-FIELD1 and R-INVK1 are quite standard. They checks the existence

Cast insertion

$$\langle \mathtt{S} \Leftarrow \mathtt{T} \rangle_{\Delta} \mathtt{e} \stackrel{\text{def}}{=} \begin{cases} \mathtt{e} & (\text{if } \Delta \vdash \mathtt{T} \precsim \mathtt{S}) \\ (\!|\mathtt{S}|\!)\,\mathtt{e} & (\text{otherwise}) \end{cases}$$

Type arguments

$$tyargs(\mathtt{C<\overline{T}>}, \mathtt{C}) = \overline{\mathtt{T}} \qquad \frac{\bullet \vdash \mathtt{N} <: \mathtt{P} \qquad tyargs(\mathtt{P}, \mathtt{C}) = \overline{\mathtt{T}}}{tyargs(\mathtt{N}, \mathtt{C}) = \overline{\mathtt{T}}}$$

Figure 5. Definition of FGJ$^{(\!|\,|\!)}$: Auxiliary functions.

$$\frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \qquad bound_{\Delta}(\mathtt{T}_0) = \mathtt{P} \qquad \Delta \vdash \mathtt{P} \precsim \mathtt{N} \qquad dynfree_{\Delta}(\mathtt{N}) \qquad mtype(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{S} \qquad \Delta \vdash \overline{\mathtt{V}} \precsim \overline{\mathtt{S}}}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) : \mathtt{S}} \text{ (Tr-Invk1)}$$

$$\frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \qquad bound_{\Delta}(\mathtt{T}_0) = \mathtt{N} \qquad \Delta \vdash \mathtt{N} \precsim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<\overline{X}>} \qquad mtype(\mathtt{m}, \mathtt{C<\overline{X}>}) = \overline{\mathtt{U}} \to \mathtt{U} \qquad \Delta \vdash \overline{\mathtt{V}} \precsim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C<\overline{X}>}](\overline{\mathtt{e}}) : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}} \text{ (Tr-Invk2)}$$

$$\frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{invoke}(\mathtt{e}_0, \mathtt{m}, \overline{\mathtt{e}}) : \mathtt{dyn<Object>}} \text{ (Tr-Invk3)} \qquad \frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{S}}{\Delta;\Gamma \vdash_{\mathrm{R}} (\!|\mathtt{T}|\!)\,\mathtt{e} : \mathtt{T}} \text{ (Tr-Cast)}$$

$$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{Error}[\mathcal{E}] : \mathtt{T} \text{ (Tr-Error)}$$

Figure 6. Definition of FGJ$^{(\!|\,|\!)}$: Typing.

$$\frac{fields(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{\mathtt{new\ N}(\overline{\mathtt{v}}).\mathtt{f}_i \longrightarrow \mathtt{v}_i} \text{ (R-Field1)} \qquad \frac{fields(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{\mathtt{get(new\ N}(\overline{\mathtt{v}})\mathtt{, f}_i\mathtt{)} \longrightarrow \mathtt{v}_i} \text{ (R-Field2)}$$

$$\frac{\bullet \vdash \mathtt{N} \precsim \mathtt{T}}{(\!|\mathtt{T}|\!)\,\mathtt{new\ N}(\overline{\mathtt{v}}) \longrightarrow \mathtt{new\ N}(\overline{\mathtt{v}})} \text{ (R-Cast)} \qquad \frac{mbody(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0}{\mathtt{new\ N}(\overline{\mathtt{v}}).\mathtt{m}(\overline{\mathtt{w}}) \longrightarrow [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N}(\overline{\mathtt{v}})/\mathtt{this}]\mathtt{e}_0} \text{ (R-Invk1)}$$

$$\frac{mbody(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0 \qquad \overline{\mathtt{w}} = \mathtt{new}\ \overline{\mathtt{P}}(\ldots) \qquad \bullet \vdash \overline{\mathtt{P}} \precsim [tyargs(\mathtt{N}, \mathtt{C})/\overline{\mathtt{X}}]\overline{\mathtt{U}}}{\mathtt{new\ N}(\overline{\mathtt{v}}).\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C<\overline{X}>}](\overline{\mathtt{w}}) \longrightarrow [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N}(\overline{\mathtt{v}})/\mathtt{this}]\mathtt{e}_0} \text{ (R-Invk2)}$$

$$\frac{mbody(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0 \qquad mtype(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}} \to \mathtt{U} \qquad \overline{\mathtt{w}} = \mathtt{new}\ \overline{\mathtt{P}}(\ldots) \qquad \bullet \vdash \overline{\mathtt{P}} \precsim \overline{\mathtt{U}}}{\mathtt{invoke(new\ N}(\overline{\mathtt{v}})\mathtt{, m, }\overline{\mathtt{w}}\mathtt{)} \longrightarrow [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N}(\overline{\mathtt{v}})/\mathtt{this}]\mathtt{e}_0} \text{ (R-Invk3)}$$

Figure 7. Definition of FGJ$^{(\!|\,|\!)}$: Reductions.

of a field/method in the receiver type but the check should never fail for well-typed expressions as we will see later. R-Field2, R-Invk2, R-Invk3 and R-Cast are for run-time checks, which can raise a run-time error. In R-Invk2, the type arguments in method parameter types are filled according to the run-time class of the receiver value, and checked against the run-time class of the actual arguments. In R-Invk3, we need to look up the method argument types by *mtype* to perform run-time checks for actual arguments. The rule R-Cast means that a cast succeeds when the subject type is run-time compatible with the target type.

We also have reduction rules for errors shown in Figure 8. Each rule has premises negating those in the corresponding reduction rule. Note that only run-time checks have error-raising reduction. We also need rules that propagate raised errors upwards; but we omit them.

FGJ$^{(\!|\,|\!)}$ is (weakly) type-safe in the sense that a well-typed program, if terminates, yield a value or raise an error. Moreover, if a program does not contain dynamic types, then it is strongly type safe.

Theorem 8 (FGJ$^{(\!|\,|\!)}$ weak type safety)**.** If $\vdash_{\mathrm{R}} (CT, \mathtt{e}) : \mathtt{T}$ and $\mathtt{e} \longrightarrow^{*} \mathtt{e}'$ where $\mathtt{e}'$ is a normal form, then $\mathtt{e}'$ is either

$$\frac{\bullet \vdash \mathtt{N} \not\precsim \mathtt{T}}{(\!|\mathtt{T}|\!)\,\mathtt{new\ N}(\overline{\mathtt{v}}) \longrightarrow \mathtt{Error[BadCast]}}\ (\text{E-CAST}) \qquad \frac{\mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \qquad \mathtt{f} \notin \overline{\mathtt{f}}}{\mathtt{get(new\ N}(\overline{\mathtt{v}})\mathtt{,f)} \longrightarrow \mathtt{Error[NoSuchField]}}\ (\text{E-FIELD})$$

$$\frac{\mathit{nomethod}(\mathtt{m},\ \mathtt{N})}{\mathtt{invoke(new\ N}(\overline{\mathtt{v}})\mathtt{,m,}\overline{\mathtt{w}}\mathtt{)} \longrightarrow \mathtt{Error[NoSuchMethod]}}\ (\text{E-INVK})$$

$$\frac{\mathit{mbody}(\mathtt{m},\ \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0 \qquad \overline{\mathtt{w}} = \mathtt{new}\ \overline{\mathtt{P}}\mathtt{(...)} \qquad \bullet \vdash \mathtt{P}_i \not\precsim [\mathit{tyargs}(\mathtt{N},\ \mathtt{C})/\overline{\mathtt{X}}]\mathtt{U}_i}{\mathtt{new\ N}(\overline{\mathtt{v}})\mathtt{.m[}\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>]}(\overline{\mathtt{w}}) \longrightarrow \mathtt{Error[IllegalArgument]}}\ (\text{E-INVK-ARG1})$$

$$\frac{\mathit{mbody}(\mathtt{m},\ \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0 \qquad \mathit{mtype}(\mathtt{m},\ \mathtt{N}) = \overline{\mathtt{U}} \to \mathtt{U} \qquad \overline{\mathtt{w}} = \mathtt{new}\ \overline{\mathtt{P}}\mathtt{(...)} \qquad \bullet \vdash \mathtt{P}_i \not\precsim \mathtt{U}_i}{\mathtt{invoke(new\ N}(\overline{\mathtt{v}})\mathtt{,m,}\overline{\mathtt{w}}\mathtt{)} \longrightarrow \mathtt{Error[IllegalArgument]}}\ (\text{E-INVK-ARG2})$$

Figure 8. Definition of FGJ$^{(\!|\,|\!)}$: Error-raising reductions.

1. a value v with $\bullet; \bullet \vdash_{\mathrm{R}} \mathtt{v} : \mathtt{N}$ and $\bullet \vdash \mathtt{N} \precsim \mathtt{T}$,
2. an error $\mathtt{Error}[\mathcal{E}]$.

Theorem 9 (FGJ$^{(\!|\,|\!)}$ strong type safety)**.** If $\vdash_{\mathrm{R}} (CT, \mathtt{e}) : \mathtt{T}$ where $(CT, \mathtt{e})$ does not contain run-time checks and $\mathtt{e} \longrightarrow^* \mathtt{e}'$ where $\mathtt{e}'$ is a normal form, then $\mathtt{e}'$ is a value v with $\bullet; \bullet \vdash_{\mathrm{R}} \mathtt{v} : \mathtt{N}$ and $\bullet \vdash \mathtt{N} \precsim \mathtt{T}$.

These theorems are proved in a standard manner of combining subject reduction and progress [34]. The statements and proofs of both properties are, in fact, very similar to those for FGJ. One non-trivial property required is transitivity of $\precsim$.

4.2 Translation from FGJ$^{\mathtt{dyn}}$ to FGJ$^{(\!|\,|\!)}$

A judgment of translation from FGJ$^{\mathtt{dyn}}$ to FGJ$^{(\!|\,|\!)}$ is of the form $\Delta; \Gamma \vdash \mathtt{e} \rightsquigarrow \mathtt{e}' : \mathtt{T}$, read "FGJ$^{\mathtt{dyn}}$ expression e of type T under environments Γ and Δ translates to FGJ$^{(\!|\,|\!)}$ expression $\mathtt{e}'$." The translation is directed by typing in FGJ$^{\mathtt{dyn}}$. We show only the rules for method invocations in Figure 9.

TRNS-INVK1 is for ordinary invocations; casts are inserted for testing run-time compatibility of arguments. If $\overline{\mathtt{V}}$ are dyn-free, then actually no casts will be inserted, thanks to Lemma 5. TRNS-INVK2 is for invocations whose arguments must be checked at run time. Note that the receiver type $\mathtt{T}_0$ in these two rules can be dyn<N> when there is an appropriate method m in N because the existence of m is statically guaranteed. TRNS-INVK3 is for invocations whose receiver type is dyn<N> and N has no appropriate method m.

Although we omit its definition, we write $(CT, \mathtt{e}) \rightsquigarrow (CT', \mathtt{e}')$ to mean that the FGJ$^{\mathtt{dyn}}$ program $(CT, \mathtt{e})$ is translated to the FGJ$^{(\!|\,|\!)}$ program $(CT', \mathtt{e}')$. Then, the translation preserves well-typedness, i.e., a well-typed FGJ$^{\mathtt{dyn}}$ program translates to a well-typed FGJ$^{(\!|\,|\!)}$ program.

Theorem 10 (Weak translation)**.** If $\vdash_{\mathrm{G}} (CT, \mathtt{e}) : \mathtt{T}$, then $(CT, \mathtt{e}) \rightsquigarrow (CT', \mathtt{e}')$ and $\vdash_{\mathrm{R}} (CT', \mathtt{e}') : \mathtt{T}$ for some $(CT', \mathtt{e}')$.

Theorem 11 (Strong translation)**.** If $\overline{\kappa} = \overline{\Diamond}$ for every $\mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N\ \{...\}}$ in CT and none of dyn<P> appears in $(CT, \mathtt{e})$ and $\vdash_{\mathrm{G}} (CT, \mathtt{e}) : \mathtt{T}$, then $(CT, \mathtt{e}) \rightsquigarrow (CT', \mathtt{e}')$ and $\vdash_{\mathrm{R}} (CT', \mathtt{e}') : \mathtt{T}$ for some $(CT', \mathtt{e}')$ and $(CT', \mathtt{e}')$ does not contain run-time checks.

Combining Theorem 8 and Theorem 10, we can see that a member access which translates to an ordinary member access without run-time checks will not fail. In other words, a statically typed portion (method invocations whose receiver and argument types are dyn-free) will never fail. That is the type safety of FGJ$^{\mathtt{dyn}}$.

5. Implementation

In this section, we report the basic implementation scheme for Java. Our plan is to add a new compilation phase to transform the code tree to have the run-time checks inserted as the same way as the other existing compilation phases such as the type erasing transformation. The transformation follows the rules in Section 4 and the run-time checks can be implemented using existing reflective features of Java. Since the current JVM has no run-time information of type arguments of generics, we need a mechanism to look up full run-time type information. We follow the technique of type passing [30–32] for this.

5.1 Run-time Checks

The run-time checks $(\!|\mathtt{T}|\!)\,\mathtt{e}$, $\mathtt{get(e,f)}$ and $\mathtt{invoke(e,m,}\overline{\mathtt{e}}\mathtt{)}$ can be implemented by reflection APIs such as java.lang.Class, java.lang.reflect.Field and java.lang.reflect.Method. We implement a class Cla to represent type descriptors and a static method Cla.$() to get the run-time type information of obj including type arguments. The return value of Cla.$(...) is a type descriptor d (an instance of class Cla described in more details later), which has field cl of a java.lang.Class instance, field p of an array of type descriptors of type

$$\frac{\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}_0' : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{V}} \qquad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N} \\ \mathit{dynfree}_\Delta(\mathtt{N}) \qquad \mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{S} \qquad \Delta \vdash \overline{\mathtt{V}} \lesssim \overline{\mathtt{S}}}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \rightsquigarrow \mathtt{e}_0'.\mathtt{m}(\langle \overline{\mathtt{S}} \Leftarrow \overline{\mathtt{V}} \rangle_\Delta \overline{\mathtt{e}}') : \mathtt{S}} \ (\textsc{Trns-Invk1})$$

$$\frac{\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}_0' : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{V}} \qquad \mathit{bound}_\Delta(\mathtt{T}_0) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<}\overline{\mathtt{X}}\mathtt{>} \\ \neg \mathit{dynfree}_\Delta(\mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) \qquad \mathit{mtype}(\mathtt{m}, \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U} \qquad \Delta \vdash \overline{\mathtt{V}} \lesssim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \rightsquigarrow \mathtt{e}_0'.\mathtt{m}[\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>}](\langle [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \Leftarrow \overline{\mathtt{V}} \rangle_\Delta \overline{\mathtt{e}}') : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}} \ (\textsc{Trns-Invk2})$$

$$\frac{\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}_0' : \mathtt{dyn<N>} \qquad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{V}} \qquad \mathit{nomethod}(\mathtt{m}, \mathtt{N})}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \rightsquigarrow \mathtt{invoke}(\mathtt{e}_0', \mathtt{m}, \overline{\mathtt{e}}') : \mathtt{dyn<Object>}} \ (\textsc{Trns-Invk3})$$

Figure 9. Translation from FGJ$^{\mathtt{dyn}}$ to FGJ$^{(\!(\,)\!)}$.

arguments, and `h` of an array of a superclass chain, where `d.h[0]` is `d` itself and `d.h[d.h.length-1]` is `Object`.

The cast $(\!(\mathtt{T})\!)\,\mathtt{e}$, which corresponds to R-CAST, can be implemented as the following method `cast()`, which casts `obj` to class `klass`.

```
Object cast(Cla klass, Object obj) {
  if ((obj instanceof Parametric &&
       isRuntimeCompatible(Cla.$(obj), klass)) ||
      klass.cl.isInstance(obj)) {
    return obj;
  } else throw new ClassCastException();
}
```

If the type of `obj` has type arguments, then `isRuntimeCompatible()` method checks if the type of `obj` and the target type of the cast satisfy the relation $\nprec$. Otherwise, the cast acts as a normal one, which uses the subtype relation $<:$. The argument type checking for $\mathtt{e.m[}\overline{\mathtt{T}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>]}(\overline{\mathtt{e}})$ in R-INVK2 can also be done by this method. Although not proved, we conjecture that the explicit transitivity rule for $\prec$ is actually redundant. Without transitivity, it is easy to check run-time compatibility.

Since $(\!(\mathtt{T})\!)$ is inserted by the translation and types $\overline{\mathtt{T}}$ in $\mathtt{e.m[}\overline{\mathtt{T}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>]}(\overline{\mathtt{e}})$ are specified by the translation, the target type of a cast can be determined mostly at compile time, except the type arguments for $\overline{\mathtt{X}}$ have to be filled at run time.

The evaluation of `get(e,f)`, which corresponds to R-FIELD2, is quite easy. We have `Object.getClass()` to get an instance of `java.lang.Class`, which has `getField()` method. `getField()` looks up a field by a field name and throws `NoSuchField` exception when no field is found. The return value of `getField()` is an instance of `java.lang.reflect.Field`, which has `get()` method, which retrieves the field value from the receiver.

```
Object get(Object r, String f) {
  Field fld = r.getClass().getField(f);
  return fld.get(r);
}
```

The evaluation of $\mathtt{invoke(e,m,}\overline{\mathtt{e}}\mathtt{)}$, which corresponds to R-INVK3, can be implemented as the method `invoke()` below. Suppose `Met` is a class for parameter types of a method, which has field `m` of `java.lang.reflect.Method` and field `params` of method type descriptors. Also suppose `mtypes()` is a method to look up method signatures by a method name.

```
Object invoke(Object r, String m, Object[] args) {
  Met[] mets = mtypes(r, m);
  M: for (int i=0, l=mets.length; i < l; i++) {
    if (mets[i].args.length != args.length)
      continue M;
    for (int j=args.length-1; 0 <= j; j--) {
      try {
        mets[i].params[j].cast(args[j]);
      } catch (ClassCastException e) {
        continue M;
      }
    }
    return mets[i].m.invoke(r, args);
  }
  throw new NoSuchMethodException();
}
```

`invoke` looks up methods named `m`, and call the first method[5] whose parameter types match the argument types. `cast()` is used to match parameters and arguments. If the casts succeed, then `java.lang.Method.invoke` API performs real invocation of the method. Otherwise, it throws `NoSuchMethodException`. Note that the target type of the cast must be determined at run time because we have no information of the receiver type at compile time.

5.2 Information on Type Arguments at Run Time

As we have already seen, we need information on type arguments at run time. It requires additional memory usage and time costs, but relatively high performance technique is proposed by Viroli et al. [30–32]. We quickly review this technique.

[5] We are not concerning method overloading here for simplicity.

The basic idea of this technique is to pass type information as a field of an object by transforming the code. For example, the following code describes how the transformation goes.

```
Cell<Shape> c
  = new Cell<Shape>(new Rectangle(...));
// Cell<Shape> c
//   = new Cell<Shape>(
//       new Cla(Cell.class,
//               new Cla[]{
//                 new Cla(Shape.class) }),
//       new Rectangle(...));
```

The first line is the original code, and the comment is the translated code. The instance of `Cla`, which is a type descriptor class, is passed as a first argument of the constructor. Of course, the definition of class `Cell` is also transformed to receive a type descriptor at the constructor, and to implement an interface, which provides access to the type descriptor.

The transformation described above is not optimized at all: a new type descriptor is generated every time the constructor is called. So, a mechanism to reduce the number of generations of type descriptors to once for a distinct type, using double hashing, is reported in [30–32].

5.3 Preliminary Benchmark Evaluation

Since we have no working compiler yet, we only give benchmarks for each run-time check with minimal hand-translated code using those checks separately. These may help to see if our implementation idea is reasonable and how dynamic types slow down execution of the program. Execution of dynamically typed code is quite expensive especially for method invocations on a receiver of dynamic type, but we believe that the cost is not unacceptable.

Benchmarks for each run-time check is shown in Table 3. For each run-time check, we used test code including a single expression with a run-time check and the same expression without it. Each test code iterates 100/10000/1000000 times and overall time consumptions are listed.

# of iterations	100	10000	1000000
$(\!\lvert \mathtt{T} \rvert\!)\,\mathtt{e}$	0.006	0.019	0.125
`e`	0.004	0.008	0.087
`get(e,f)`	0.003	0.043	0.356
`e.f`	0.001	0.003	0.056
$\mathtt{e.m[}\overline{\mathtt{T}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>](}\overline{\mathtt{e}}\mathtt{)}$	0.000	0.003	0.072
$\mathtt{e.m(}\overline{\mathtt{e}}\mathtt{)}$	0.000	0.002	0.080
$\mathtt{invoke(e,m,}\overline{\mathtt{e}}\mathtt{)}$	0.007	0.093	3.430
$\mathtt{e.m(}\overline{\mathtt{e}}\mathtt{)}$	0.000	0.003	0.069

Table 3. Execution time of each run-time check (sec.)

For the expression $(\!\lvert \mathtt{T} \rvert\!)\,\mathtt{e}$, we used a static type for the target type of the cast. The cast looks into type arguments but we can say it is not so expensive according to the result. We can say the same thing about the run-time check in the expression $\mathtt{e.m[}\overline{\mathtt{T}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>](}\overline{\mathtt{e}}\mathtt{)}$ since it only needs a cast for each argument. The run-time check in the expression `get(e,f)` is relatively expensive because it uses a feature of `java.lang.reflect.Field`. The run-time check in the expression $\mathtt{invoke(e,m,}\overline{\mathtt{e}}\mathtt{)}$ is quite expensive but the cost seems somewhat inevitable cost since it must resolve a method signature and check run-time types of the arguments.

In Table 4, we have tested casts with more complex target types. We used a dynamic type argument for the target type. The dynamic type argument has a bound, which is a class type possibly including another dynamic type argument. We count the nested dynamic type argument as depth and the result for each target type of the depth is listed in the rows.

Depth(s) \ # of iterations	100	10000	1000000
1	0.000	0.004	0.105
2	0.000	0.007	0.171
3	0.001	0.009	0.235
4	0.001	0.008	0.268
5	0.001	0.009	0.301

Table 4. Execution time of casts (sec.)

We can conclude that the operation for checking the run-time compatibility $\asymp$ is not too expensive in comparison with the other run-time checks.

6. Related Work

There is much work on mixing dynamic and static types (see, for example, Siek and Taha [25, 26] for a more extensive survey). Here, we compare our work mainly with related work on object-oriented programming languages and parametric polymorphism.

We first review proposals to apply static type checking to dynamically typed languages. Bracha and Griswold [7] have proposed *Strongtalk*, which is a typechecker for a downward compatible variant of Smalltalk, a dynamically typed class-based object-oriented programming language. The type system of *Strongtalk* is structural and supports subtyping and generics but does not accept partially typed programs. Thiemann [28] has proposed a type system for (a subset of) JavaScript, which is a prototype-based object-oriented language, to avoid some kind of run-time errors by static type checking. Furr, An, Foster, and Hicks [12] have developed Diamondback Ruby, an extension of Ruby with a static type system. Their type system, which seems useful to find bugs, however, does not offer static type safety.

Anderson and Drossopoulou [3] have proposed a type system for (a subset of) JavaScript for the evolution from JavaScript to Java. Although it is nominal and concerned about script-to-program evolution, their type system does not have subtyping, inheritance, or polymorphism; more-

over, this work is not concerned about safety of partially typed programs in the middle of the evolution.

Lagorio and Zucca [21] have developed Just, an extension of Java with unknown types. Although there is some overlap in the expected uses of this system and gradual typing, the main purpose of unknown types is to omit type declarations; possibly unsafe use of unknown types is rejected by the type system. They use reflection to implement member access on unknown types.

Gray, Findler, and Flatt [13] have implemented an extension of Java with dynamic types and contract checking [10] for interoperability with Scheme. They mainly focus on the design and implementation issues and give no discussion on the interaction with generics. Their technique to implement reflective calls can be used for our setting.

As we have already mentioned, Siek and Taha have studied gradual typing for Abadi-Cardelli's object calculus [26]. However, the language is object-based (as opposed to class-based) and parametric polymorphism is not studied. Another point is that the implementation of run-time checks for class-based languages seems easier than that for object-based languages, since, in class-based languages, every value is tagged with its run-time type information and the check can be performed in one step (unlike higher-order contract checking, which checks inputs to and outputs from functions separately).

Sage [14], a functional language based on hybrid type checking [11], supports both parametric polymorphism and dynamic types. Matthews and Ahmed [23] and, more recently, Ahmed, Findler, Siek, and Wadler [2] give theoretical accounts for the combination of impredicative polymorphism with dynamic typing. In all of these works, a dynamic type is compatible (in our terminology) with universal types whereas there is no counterpart of universal types in our setting. None of them has addressed *bounded* polymorphism.

Wrigstad, Nardelli, Lebresne, Östlund and Vitek [5, 35] have developed a language called Thorn, which integrates static and dynamic types in a different way. They have introduced the notion of *like types*, which interface between statically and dynamically typed code. A variable of `like C` is treated as type `C` at compile time but any value can flow into the variable at run time (subject to run-time checks). This is different from `dyn<C>`, which allows any operations statically but only subtypes of `C` can flow into.

Bierman, Meijer and Torgersen[4] added dynamic types to C♯. They also translate a program of the surface language into intermediate code, which has explicit run-time checks. In their setting, dynamic types can be arguments of generic class, but their subtype relation is only invariant with respect to type parameters, so, for example, it is not possible to pass `Cell<Rectangle>` to `Cell<dyn>`.

There are some related features already exist in Java. Wildcards studied by Igarashi and Viroli [17] and Torgersen, Ernst, Hansen, Ahé, Bracha and Gafter [29] enable a flexible subtyping with both co- and contra-variant parametric types, though only statically resolved members can be accessed on a receiver of a wildcard type and it is not allowed to specify a wildcard type as a type argument in a `new` expression. Raw types are proposed by Bracha, Odersky, Stoutamire, and Wadler [6] to deal with compatibility between legacy monomorphic Java code and new polymorphic Java code. A generic class $\texttt{C<}\overline{\texttt{X}}\texttt{>}$ can be used as a raw type `C`, without type arguments, and assigning a value of $\texttt{C<}\overline{\texttt{T}}\texttt{>}$ to a variable of `C`, or even creating an instance of `C` via `new` expression are allowed. This behavior is similar to FGJ$^{\texttt{dyn}}$'s if we consider `C` to be an abbreviation of `C<dyn, ..., dyn>`. However, with raw types, even statically typed code can go wrong.

7. Conclusions

We have designed a language, which combines dynamic types and generics. The language allows dynamic types to be used as type arguments of a generic class and realizes smooth interfacing between dynamically and statically typed code thanks to the flexible compatibility relation. We have introduced bounded dynamic types to deal with the case where a type parameter with an upper bound.

The language is formalized as a minimal core model of Java including the feature of generics. As in other gradual type systems, we have proved safety properties, which ensure that statically typed parts in a program never go wrong.

We have also reviewed the sketch of implementation scheme, which is an idea to develop a gradually typed Java compiler by extending an existing Java compiler without modifying JVM.

Future Work. Our run-time compatibility does not allow argument passing, for example, from `Cell<dyn>` to `Cell<Rectangle>`. It might be too early to abort the execution at this point since the value in the `Cell` may not be used at all. We think we can relax the restriction by deferring the check until the field is accessed. Then, we need a blame assignment system [2, 33] for precise error reports.

When a library bytecode of a generic class compiled by the standard Java compiler is used with a client bytecode compiled by our compiler, the client is not allowed to use dynamic type arguments to the generic class since type variables in the generic class are declared without kind ♦. We think that converting the library bytecode at load time helps to relax this restriction.

We also plan to investigate the interactions of dynamic types with other features such as overloading to make the language more realistic.

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A. Definitions

Field lookup

$$fields(\texttt{Object}) = \bullet \qquad \frac{\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{> } \lhd \texttt{ N }\{\overline{\texttt{S}}\ \overline{\texttt{f}}\texttt{; ... }\} \qquad fields([\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}) = \overline{\texttt{U}}\ \overline{\texttt{g}}}{fields(\texttt{C<}\overline{\texttt{T}}\texttt{>}) = \overline{\texttt{U}}\ \overline{\texttt{g}}\texttt{, } [\overline{\texttt{T}}/\overline{\texttt{X}}]\overline{\texttt{S}}\ \overline{\texttt{f}}}$$

Method type lookup

$$\frac{\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{> } \lhd \texttt{ N }\{\ldots\texttt{; }\ \overline{\texttt{M}}\texttt{}\} \qquad \texttt{ U m(}\overline{\texttt{U}}\ \overline{\texttt{x}}\texttt{)\{ return e; }\} \in \overline{\texttt{M}}}{mtype(\texttt{m}, \texttt{C<}\overline{\texttt{T}}\texttt{>}) = [\overline{\texttt{T}}/\overline{\texttt{X}}](\overline{\texttt{U}} \to \texttt{U})} \qquad \frac{\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{> } \lhd \texttt{ N }\{\ldots\texttt{; }\ \overline{\texttt{M}}\texttt{}\} \qquad \texttt{m} \notin \overline{\texttt{M}} \qquad mtype(\texttt{m}, [\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}) = \overline{\texttt{U}} \to \texttt{U}}{mtype(\texttt{m}, \texttt{C<}\overline{\texttt{T}}\texttt{>}) = \overline{\texttt{U}} \to \texttt{U}}$$

$$nomethod(\texttt{m}, \texttt{Object}) \qquad \frac{\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{> } \lhd \texttt{ N }\{\ldots\texttt{; }\ \overline{\texttt{M}}\texttt{}\} \qquad \texttt{m} \notin \overline{\texttt{M}} \qquad nomethod(\texttt{m}, [\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N})}{nomethod(\texttt{m}, \texttt{C<}\overline{\texttt{T}}\texttt{>})}$$

Figure 10. Definition of FGJ$^{\texttt{dyn}}$: Auxiliary functions and predicates.

Method body lookup

$$\frac{\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{> } \lhd \texttt{ N }\{\ldots\texttt{; }\ \overline{\texttt{M}}\texttt{}\} \qquad \texttt{ U m(}\overline{\texttt{U}}\ \overline{\texttt{x}}\texttt{)\{ return e; }\} \in \overline{\texttt{M}}}{mbody(\texttt{m}, \texttt{C<}\overline{\texttt{T}}\texttt{>}) = \overline{\texttt{x}}.[\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{e}} \qquad \frac{\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}} \lhd \overline{\texttt{N}}\texttt{> } \lhd \texttt{ N }\{\ldots\texttt{; }\ \overline{\texttt{M}}\texttt{}\} \qquad \texttt{m} \notin \overline{\texttt{M}} \qquad mbody(\texttt{m}, [\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}) = \overline{\texttt{x}}.\texttt{e}}{mbody(\texttt{m}, \texttt{C<}\overline{\texttt{T}}\texttt{>}) = \overline{\texttt{x}}.\texttt{e}}$$

Figure 11. Definition of FGJ$^{()}$: Auxiliary functions and predicates.

Expression typing

$$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{x} : \Gamma(\mathtt{x}) \ \text{(Tr-Var)} \qquad \frac{\Delta \vdash \mathtt{N}\ \mathrm{ok} \qquad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \qquad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{U}} \qquad \Delta \vdash \overline{\mathtt{U}} \prec \overline{\mathtt{T}}}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{new}\ \mathtt{N}(\overline{\mathtt{e}}) : \mathtt{N}} \ \text{(Tr-New)}$$

$$\frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \qquad \mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0.\mathtt{f}_i : \mathtt{T}_i} \ \text{(Tr-Field1)} \qquad \frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{get}(\mathtt{e}_0, \mathtt{f}) : \mathtt{dyn<Object>}} \ \text{(Tr-Field2)}$$

$$\frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \qquad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{P} \qquad \Delta \vdash \mathtt{P} \prec \mathtt{N} \qquad \mathit{dynfree}_\Delta(\mathtt{N}) \qquad \mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{S} \qquad \Delta \vdash \overline{\mathtt{V}} \prec \overline{\mathtt{S}}}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) : \mathtt{S}} \ \text{(Tr-Invk1)}$$

$$\frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \qquad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N} \qquad \Delta \vdash \mathtt{N} \prec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<}\overline{\mathtt{X}}\mathtt{>} \qquad \mathit{mtype}(\mathtt{m}, \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U} \qquad \Delta \vdash \overline{\mathtt{V}} \prec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>}](\overline{\mathtt{e}}) : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}} \ \text{(Tr-Invk2)}$$

$$\frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}}{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{invoke}(\mathtt{e}_0, \mathtt{m}, \overline{\mathtt{e}}) : \mathtt{dyn<Object>}} \ \text{(Tr-Invk3)} \qquad \frac{\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{S}}{\Delta;\Gamma \vdash_{\mathrm{R}} (\!|\mathtt{T}|\!)\,\mathtt{e} : \mathtt{T}} \ \text{(Tr-Cast)}$$

$$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{Error}[\mathcal{E}] : \mathtt{T} \ \text{(Tr-Error)}$$

Method typing

$$\frac{\Delta = \overline{\mathtt{X}^{\kappa}} <: \overline{\mathtt{N}} \qquad \Delta \vdash \overline{\mathtt{T}}, \mathtt{T}\ \mathrm{ok} \qquad \Delta; \overline{\mathtt{x}} : \overline{\mathtt{T}}, \mathtt{this} : \mathtt{C<}\overline{\mathtt{X}}\mathtt{>} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{S} \qquad \Delta \vdash \mathtt{S} \prec \mathtt{T} \qquad \mathtt{class}\ \mathtt{C<}\overline{\mathtt{X}^{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \{\ldots\} \qquad \mathit{override}_\Delta(\mathtt{m}, \mathtt{N}, \overline{\mathtt{T}} \to \mathtt{T})}{\mathtt{T}\ \mathtt{m}(\overline{\mathtt{T}}\ \overline{\mathtt{x}})\{\ \mathtt{return}\ \mathtt{e}_0;\ \}\ \mathtt{OK\ IN}\ \mathtt{C<}\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\mathtt{>}} \ \text{(Tr-Method)}$$

Class typing

$$\frac{\forall \mathtt{N}_i \in \overline{\mathtt{N}}.(\mathtt{X}_1^{\kappa_1} <: \mathtt{N}_1, \ldots, \mathtt{X}_{i-1}^{\kappa_{i-1}} <: \mathtt{N}_{i-1} \vdash \mathtt{N}_i\ \mathrm{ok}) \qquad \overline{\mathtt{X}^{\kappa}} <: \overline{\mathtt{N}} \vdash \mathtt{N}, \overline{\mathtt{T}}\ \mathrm{ok} \qquad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{U}}\ \overline{\mathtt{g}} \qquad \overline{\mathtt{M}}\ \mathtt{OK\ IN}\ \mathtt{C<}\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\mathtt{>} \qquad \mathtt{K} = \mathtt{C}(\overline{\mathtt{U}}\ \overline{\mathtt{g}}, \overline{\mathtt{T}}\ \overline{\mathtt{f}})\{\mathtt{super}(\overline{\mathtt{g}});\ \mathtt{this}.\overline{\mathtt{f}}=\overline{\mathtt{f}};\}}{\mathtt{class}\ \mathtt{C<}\overline{\mathtt{X}^{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\ \mathtt{K}\ \overline{\mathtt{M}}\}\ \mathtt{OK}} \ \text{(Tr-Class)}$$

Figure 12. Definition of FGJ$^{(\!|\,|\!)}$: Typing.

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{e}_0'}{\mathtt{e}_0.\mathtt{f} \longrightarrow \mathtt{e}_0'.\mathtt{f}}\ (\text{RC-Field1}) \qquad \frac{\mathtt{e}_0 \longrightarrow \mathtt{e}_0'}{\mathtt{get}(\mathtt{e}_0, \mathtt{f}) \longrightarrow \mathtt{get}(\mathtt{e}_0', \mathtt{f})}\ (\text{RC-Field2})$$

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{e}_0'}{\mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \longrightarrow \mathtt{e}_0'.\mathtt{m}(\overline{\mathtt{e}})}\ (\text{RC-Invk-Recv1}) \qquad \frac{\mathtt{e} \longrightarrow \mathtt{e}'}{\mathtt{v}_0.\mathtt{m}(\overline{\mathtt{v}}, \mathtt{e}, \overline{\mathtt{e}}) \longrightarrow \mathtt{v}_0.\mathtt{m}(\overline{\mathtt{v}}, \mathtt{e}', \overline{\mathtt{e}})}\ (\text{RC-Invk-Arg1})$$

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{e}_0'}{\mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\langle\overline{\mathtt{X}}\rangle](\overline{\mathtt{e}}) \longrightarrow \mathtt{e}_0'.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\langle\overline{\mathtt{X}}\rangle](\overline{\mathtt{e}})}\ (\text{RC-Invk-Recv2})$$

$$\frac{\mathtt{e} \longrightarrow \mathtt{e}'}{\mathtt{v}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\langle\overline{\mathtt{X}}\rangle](\overline{\mathtt{v}}, \mathtt{e}, \overline{\mathtt{e}}) \longrightarrow \mathtt{v}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\langle\overline{\mathtt{X}}\rangle](\overline{\mathtt{v}}, \mathtt{e}', \overline{\mathtt{e}})}\ (\text{RC-Invk-Arg2})$$

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{e}_0'}{\mathtt{invoke}(\mathtt{e}_0, \mathtt{m}, \overline{\mathtt{e}}) \longrightarrow \mathtt{invoke}(\mathtt{e}_0', \mathtt{m}, \overline{\mathtt{e}})}\ (\text{RC-Invk-Recv3})$$

$$\frac{\mathtt{e} \longrightarrow \mathtt{e}'}{\mathtt{invoke}(\mathtt{v}_0, \mathtt{m}, \overline{\mathtt{v}}, \mathtt{e}, \overline{\mathtt{e}}) \longrightarrow \mathtt{invoke}(\mathtt{v}_0, \mathtt{m}, \overline{\mathtt{v}}, \mathtt{e}', \overline{\mathtt{e}})}\ (\text{RC-Invk-Arg3})$$

$$\frac{\mathtt{e} \longrightarrow \mathtt{e}'}{\mathtt{new}\ \mathtt{N}(\overline{\mathtt{v}}, \mathtt{e}, \overline{\mathtt{e}}) \longrightarrow \mathtt{new}\ \mathtt{N}(\overline{\mathtt{v}}, \mathtt{e}', \overline{\mathtt{e}})}\ (\text{RC-New-Arg}) \qquad \frac{\mathtt{e}_0 \longrightarrow \mathtt{e}_0'}{(\!|\mathtt{T}|\!)\,\mathtt{e}_0 \longrightarrow (\!|\mathtt{T}|\!)\,\mathtt{e}_0'}\ (\text{RC-Cast})$$

Figure 13. Definition of FGJ$^{(\!|\,|\!)}$: Reductions (congruence).

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{e}_0.\mathtt{f} \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Field1}) \qquad \frac{\mathtt{e}_0 \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{get}(\mathtt{e}_0, \mathtt{f}) \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Field2})$$

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Invk-Recv1}) \qquad \frac{\mathtt{e} \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{v}_0.\mathtt{m}(\overline{\mathtt{v}}, \mathtt{e}, \overline{\mathtt{e}}) \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Invk-Arg1})$$

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\langle\overline{\mathtt{X}}\rangle](\overline{\mathtt{e}}) \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Invk-Recv2}) \qquad \frac{\mathtt{e} \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{v}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\langle\overline{\mathtt{X}}\rangle](\overline{\mathtt{v}}, \mathtt{e}, \overline{\mathtt{e}}) \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Invk-Arg2})$$

$$\frac{\mathtt{e}_0 \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{invoke}(\mathtt{e}_0, \mathtt{m}, \overline{\mathtt{e}}) \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Invk-Recv3}) \qquad \frac{\mathtt{e} \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{invoke}(\mathtt{v}_0, \mathtt{m}, \overline{\mathtt{v}}, \mathtt{e}, \overline{\mathtt{e}}) \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Invk-Arg3})$$

$$\frac{\mathtt{e} \longrightarrow \mathtt{Error}[\mathcal{E}]}{\mathtt{new}\ \mathtt{N}(\overline{\mathtt{v}}, \mathtt{e}, \overline{\mathtt{e}}) \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-New-Arg}) \qquad \frac{\mathtt{e}_0 \longrightarrow \mathtt{Error}[\mathcal{E}]}{(\!|\mathtt{T}|\!)\,\mathtt{e}_0 \longrightarrow \mathtt{Error}[\mathcal{E}]}\ (\text{EC-Cast})$$

Figure 14. Definition of FGJ$^{(\!|\,|\!)}$: Error-propagating reductions.

Expression translation

$$\Delta;\Gamma \vdash \mathtt{x} \rightsquigarrow \mathtt{x} : \Gamma(\mathtt{x}) \ \text{(Trns-Var)}$$

$$\frac{\Delta \vdash \mathtt{N}\ \mathrm{ok} \qquad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \qquad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}'} : \overline{\mathtt{U}} \qquad \Delta \vdash \overline{\mathtt{U}} \lesssim \overline{\mathtt{T}}}{\Delta;\Gamma \vdash \mathtt{new\ N(\overline{e})} \rightsquigarrow \mathtt{new\ N}(\langle \overline{\mathtt{T}} \Leftarrow \overline{\mathtt{U}} \rangle_{\Delta} \overline{\mathtt{e}'}) : \mathtt{N}} \ \text{(Trns-New)}$$

$$\frac{\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{T}_0 \qquad \mathit{fields}(\mathit{bound}_{\Delta}(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{f}_i \rightsquigarrow \mathtt{e}'_0.\mathtt{f}_i : \mathtt{T}_i} \ \text{(Trns-Field1)}$$

$$\frac{\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{dyn<N>} \qquad \mathtt{f} \notin \overline{\mathtt{f}} \qquad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{f} \rightsquigarrow \mathtt{get(e}'_0\mathtt{, f)} : \mathtt{dyn<Object>}} \ \text{(Trns-Field2)}$$

$$\frac{\begin{array}{c}\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}'} : \overline{\mathtt{V}} \qquad \mathit{bound}_{\Delta}(\mathtt{T}_0) = \mathtt{N} \\ \mathit{dynfree}_{\Delta}(\mathtt{N}) \qquad \mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{S} \qquad \Delta \vdash \overline{\mathtt{V}} \lesssim \overline{\mathtt{S}}\end{array}}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m(\overline{e})} \rightsquigarrow \mathtt{e}'_0.\mathtt{m}(\langle \overline{\mathtt{S}} \Leftarrow \overline{\mathtt{V}} \rangle_{\Delta} \overline{\mathtt{e}'}) : \mathtt{S}} \ \text{(Trns-Invk1)}$$

$$\frac{\begin{array}{c}\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{T}_0 \qquad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}'} : \overline{\mathtt{V}} \qquad \mathit{bound}_{\Delta}(\mathtt{T}_0) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<\overline{X}>} \\ \neg \mathit{dynfree}_{\Delta}(\mathtt{C<\overline{T}>}) \qquad \mathit{mtype}(\mathtt{m}, \mathtt{C<\overline{X}>}) = \overline{\mathtt{U}} \to \mathtt{U} \qquad \Delta \vdash \overline{\mathtt{V}} \lesssim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}\end{array}}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m(\overline{e})} \rightsquigarrow \mathtt{e}'_0.\mathtt{m[\overline{U}|C<\overline{X}>]}(\langle [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \Leftarrow \overline{\mathtt{V}} \rangle_{\Delta} \overline{\mathtt{e}'}) : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}} \ \text{(Trns-Invk2)}$$

$$\frac{\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{dyn<N>} \qquad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}'} : \overline{\mathtt{V}} \qquad \mathit{nomethod}(\mathtt{m}, \mathtt{N})}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m(\overline{e})} \rightsquigarrow \mathtt{invoke(e}'_0\mathtt{, m, \overline{e}')} : \mathtt{dyn<Object>}} \ \text{(Trns-Invk3)}$$

Method translation

$$\frac{\begin{array}{c}\Delta = \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}} \qquad \Delta \vdash \overline{\mathtt{T}}, \mathtt{T}\ \mathrm{ok} \qquad \Delta; \overline{\mathtt{x}} : \overline{\mathtt{T}}, \mathtt{this} : \mathtt{C<\overline{X}>} \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{S} \\ \Delta \vdash \mathtt{S} \lesssim \mathtt{T} \qquad \mathtt{class\ C<\overline{X}^{\overline{\kappa}} \lhd \overline{N}> \lhd N\ \{\ldots\}} \qquad \mathit{override}_{\Delta}(\mathtt{m}, \mathtt{N}, \overline{\mathtt{T}} \to \mathtt{T})\end{array}}{\mathtt{T\ m(\overline{T}\ \overline{x})\{\ return\ e_0;\ \}} \rightsquigarrow \mathtt{T\ m(\overline{T}\ \overline{x})\{\ return\ \langle T \Leftarrow S \rangle_{\Delta} e'_0;\ \}\ IN\ C<\overline{X} \lhd \overline{N}>}}$$

Class translation

$$\frac{\begin{array}{c}\forall \mathtt{N}_i \in \overline{\mathtt{N}}.(\mathtt{X}_1^{\kappa_1} <: \mathtt{N}_1, \ldots, \mathtt{X}_{i-1}^{\kappa_{i-1}} <: \mathtt{N}_{i-1} \vdash \mathtt{N}_i\ \mathrm{ok}) \qquad \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}} \vdash \mathtt{N}, \overline{\mathtt{T}}\ \mathrm{ok} \\ \mathit{fields}(\mathtt{N}) = \overline{\mathtt{U}}\ \overline{\mathtt{g}} \qquad \overline{\mathtt{M}} \rightsquigarrow \overline{\mathtt{M}}'\ \mathtt{IN\ C<\overline{X} \lhd \overline{N}>} \qquad \mathtt{K} = \mathtt{\ C(\overline{U}\ \overline{g}, \overline{T}\ \overline{f})\{super(\overline{g}); this.\overline{f}=\overline{f};\}}\end{array}}{\mathtt{class\ C<\overline{X}^{\overline{\kappa}} \lhd \overline{N}> \lhd N\ \{\overline{T}\ \overline{f};\ K\ \overline{M}\}} \rightsquigarrow \mathtt{class\ C<\overline{X}^{\overline{\kappa}} \lhd \overline{N}> \lhd N\ \{\overline{T}\ \overline{f};\ K\ \overline{M}'\}}}$$

Figure 15. Translation from FGJ$^{\mathtt{dyn}}$ to FGJ$^{(\!(\cdot)\!)}$.

B. Proof of Properties

B.1 Compatibility

We use the following mapping $depth$ from a type to an integer for proofs.

$$
\begin{array}{lcl}
depth_\Delta(\texttt{Object}) & \stackrel{\text{def}}{=} & 0 \\
depth_\Delta(\texttt{C<}\overline{\texttt{T}}\texttt{>}) & \stackrel{\text{def}}{=} & depth_\Delta([\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}) + 1 \quad (\text{where } \texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}}\texttt{>} \triangleleft \overline{\texttt{N}}\texttt{>} \triangleleft \texttt{N \{...\}}) \\
depth_\Delta(\texttt{X}) & \stackrel{\text{def}}{=} & depth_\Delta(bound_\Delta(\texttt{X})) + 1 \\
depth_\Delta(\texttt{dyn<N>}) & \stackrel{\text{def}}{=} & depth_\Delta(\texttt{N}) + 1
\end{array}
$$

Lemma 12. $depth_\Delta(\texttt{C<}\overline{\texttt{T}}\texttt{>}) = depth_\Delta(\texttt{C<}\overline{\texttt{S}}\texttt{>})$ for any Δ, C<$\overline{\texttt{T}}$> and C<$\overline{\texttt{S}}$>.

Proof. By induction on the definition of $depth_\Delta(\texttt{C<}\overline{\texttt{T}}\texttt{>})$.

Case $\texttt{C<}\overline{\texttt{T}}\texttt{>} = \texttt{Object}$ ($\texttt{C<}\overline{\texttt{S}}\texttt{>} = \texttt{Object}$). $depth_\Delta(\texttt{C<}\overline{\texttt{T}}\texttt{>}) = depth_\Delta(\texttt{C<}\overline{\texttt{S}}\texttt{>}) = 0$.

Case $\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}}\texttt{>} \triangleleft \overline{\texttt{N}}\texttt{>} \triangleleft \texttt{N \{...\}}$. By the induction hypothesis, we have $depth_\Delta([\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}) = depth_\Delta([\overline{\texttt{S}}/\overline{\texttt{X}}]\texttt{N})$. Then, $depth_\Delta(\texttt{C<}\overline{\texttt{T}}\texttt{>}) = depth_\Delta(\texttt{C<}\overline{\texttt{S}}\texttt{>}) = depth_\Delta([\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}) + 1$.

□

Lemma 13. If $\Delta \vdash \texttt{S} <: \texttt{T}$ and $\texttt{S} \neq \texttt{T}$, then $depth_\Delta(\texttt{T}) < depth_\Delta(\texttt{S})$.

Proof. By induction on the derivation of $\Delta \vdash \texttt{S} <: \texttt{T}$.

Case $\dfrac{\Delta \vdash \texttt{S} <: \texttt{U} \qquad \Delta \vdash \texttt{U} <: \texttt{T}}{\Delta \vdash \texttt{S} <: \texttt{T}}$. If $\texttt{S} = \texttt{U}$ or $\texttt{T} = \texttt{U}$, then the conclusion is immediate from the induction hypothesis. Otherwise ($\texttt{S} \neq \texttt{U}$ and $\texttt{T} \neq \texttt{U}$), by the induction hypothesis, $depth_\Delta(\texttt{U}) < depth_\Delta(\texttt{S})$ and $depth_\Delta(\texttt{T}) < depth_\Delta(\texttt{U})$. Then $depth_\Delta(\texttt{T}) < depth_\Delta(\texttt{U}) < depth_\Delta(\texttt{S})$.

Case $\Delta \vdash \texttt{X} <: bound_\Delta(\texttt{X})$ (where $\texttt{S} = \texttt{X}$, $\texttt{T} = bound_\Delta(\texttt{X})$). By definition of $depth$, $depth_\Delta(\texttt{S}) = depth_\Delta(\texttt{X}) = depth_\Delta(bound_\Delta(\texttt{X})) + 1 > depth_\Delta(bound_\Delta(\texttt{X})) = depth_\Delta(\texttt{T})$.

Case $\dfrac{\texttt{class C<}\overline{\texttt{X}}^{\overline{\kappa}}\texttt{>} \triangleleft \overline{\texttt{N}}\texttt{>} \triangleleft \texttt{N \{...\}}}{\Delta \vdash \texttt{C<}\overline{\texttt{T}}\texttt{>} <: [\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}}$ (where $\texttt{S} = \texttt{C<}\overline{\texttt{S}}\texttt{>}$, $\texttt{T} = [\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}$). By definition of $depth$, $depth_\Delta(\texttt{S}) = depth_\Delta(\texttt{C<}\overline{\texttt{T}}\texttt{>}) = depth_\Delta([\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}) + 1 > depth_\Delta([\overline{\texttt{T}}/\overline{\texttt{X}}]\texttt{N}) = depth_\Delta(\texttt{T})$.

Case $\Delta \vdash \texttt{dyn<N>} <: \texttt{N}$ (where $\texttt{S} = \texttt{dyn<N>}$, $\texttt{T} = \texttt{N}$). By definition of $depth$, $depth_\Delta(\texttt{S}) = depth_\Delta(\texttt{dyn<N>}) = depth_\Delta(\texttt{N}) + 1 > depth_\Delta(\texttt{N}) = depth_\Delta(\texttt{T})$.

□

Lemma 14. If $\Delta \vdash \texttt{S} <: \texttt{T}$, then $depth_\Delta(\texttt{T}) \leq depth_\Delta(\texttt{S})$.

Proof. Immediate from Lemma 13. □

Lemma 15. If $\Delta \vdash \texttt{T} <: \texttt{X}$, then $\texttt{T} = \texttt{X}$.

Proof. By induction on the derivation of $\Delta \vdash \texttt{T} <: \texttt{X}$ with a case analysis on the last rule used.

Case $\Delta \vdash \texttt{X} <: \texttt{X}$ ($\texttt{T} = \texttt{X}$). Immediate.

Case $\dfrac{\Delta \vdash \texttt{T} <: \texttt{U} \qquad \Delta \vdash \texttt{U} <: \texttt{X}}{\Delta \vdash \texttt{T} <: \texttt{X}}$. By the induction hypothesis, we have $\texttt{U} = \texttt{X}$. Then, by the induction hypothesis, we have $\texttt{T} = \texttt{U} = \texttt{X}$.

□

Lemma 16. If $\Delta \vdash \texttt{T} <: \texttt{dyn<N>}$, then $\texttt{T} = \texttt{dyn<N>}$.

Proof. By induction on the derivation of $\Delta \vdash \texttt{T} <: \texttt{dyn<N>}$ with a case analysis on the last rule used.

Case $\Delta \vdash \texttt{dyn<N>} <: \texttt{dyn<N>}$ ($\texttt{T} = \texttt{dyn<N>}$). Immediate.

Case $\dfrac{\Delta \vdash \texttt{T} <: \texttt{U} \qquad \Delta \vdash \texttt{U} <: \texttt{dyn<N>}}{\Delta \vdash \texttt{T} <: \texttt{dyn<N>}}$. By the induction hypothesis, we have $\texttt{U} = \texttt{dyn<N>}$. Then, by the induction hypothesis, we have $\texttt{T} = \texttt{U} = \texttt{dyn<N>}$.

□

Lemma 17. If $\Delta \vdash \texttt{C<}\overline{\texttt{T}}\texttt{>} <: \texttt{C<}\overline{\texttt{S}}\texttt{>}$, then $\overline{\texttt{T}} = \overline{\texttt{S}}$.

Proof. By induction on the derivation of $\Delta \vdash \texttt{C<}\overline{\texttt{T}}\texttt{>} <: \texttt{C<}\overline{\texttt{S}}\texttt{>}$.

Case $\Delta \vdash \texttt{C<}\overline{\texttt{T}}\texttt{>} <: \texttt{C<}\overline{\texttt{T}}\texttt{>}$ ($\texttt{C<}\overline{\texttt{S}}\texttt{>} = \texttt{C<}\overline{\texttt{T}}\texttt{>}$). Immediate.

Case $\dfrac{\Delta \vdash \texttt{C<}\overline{\texttt{T}}\texttt{>} <: \texttt{U} \qquad \Delta \vdash \texttt{U} <: \texttt{C<}\overline{\texttt{S}}\texttt{>}}{\Delta \vdash \texttt{C<}\overline{\texttt{T}}\texttt{>} <: \texttt{C<}\overline{\texttt{S}}\texttt{>}}$. By Lemma 15 and Lemma 16, we have $\texttt{U} = \texttt{N}$. By the fact that the relation $<:$ does not loop, we have $\texttt{N} = \texttt{C<}\overline{\texttt{U}}\texttt{>}$. Then the conclusion is immediate from the induction hypothesis.

□

Lemma 18. If $\Delta \vdash \texttt{dyn<N>} <: \texttt{dyn<P>}$, then $\texttt{N} = \texttt{P}$.

Proof. By induction on the derivation of $\Delta \vdash \texttt{dyn<N>} <: \texttt{dyn<P>}$.

Case $\Delta \vdash \texttt{dyn<N>} <: \texttt{dyn<N>}$ ($\texttt{dyn<P>} = \texttt{dyn<N>}$). Immediate.

Case $\dfrac{\Delta \vdash \texttt{dyn<N>} <: \texttt{U} \qquad \Delta \vdash \texttt{U} <: \texttt{dyn<P>}}{\Delta \vdash \texttt{dyn<N>} <: \texttt{dyn<P>}}$. By Lemma 16, we have $\texttt{U} = \texttt{dyn<P>}$. Then the conclusion is immediate from the induction hypothesis.

□

Lemma 19. If $\Delta \vdash \texttt{dyn<C<}\overline{\texttt{T}}\texttt{>>} <: \texttt{C<}\overline{\texttt{S}}\texttt{>}$, then $\overline{\texttt{T}} = \overline{\texttt{S}}$.

Proof. We show that if $\Delta \vdash \texttt{N} <: \texttt{C<}\overline{\texttt{S}}\texttt{>}$ and $\Delta \vdash \texttt{dyn<C<}\overline{\texttt{T}}\texttt{>>} <: \texttt{N}$, then $\overline{\texttt{T}} = \overline{\texttt{S}}$, by induction on the derivation of $\Delta \vdash \texttt{dyn<C<}\overline{\texttt{T}}\texttt{>>} <: \texttt{N}$.

Case $\dfrac{\Delta \vdash \texttt{dyn<C<}\overline{\texttt{T}}\texttt{>>} <: \texttt{U} \qquad \Delta \vdash \texttt{U} <: \texttt{N}}{\Delta \vdash \texttt{dyn<C<}\overline{\texttt{T}}\texttt{>>} <: \texttt{N}}$. By Lemma 15, we have $\texttt{U} = \texttt{P}$ or $\texttt{U} = \texttt{dyn<P>}$. If $\texttt{U} = \texttt{dyn<P>}$, then we have $\texttt{P} = \texttt{C<}\overline{\texttt{T}}\texttt{>}$ by Lemma 16 and the conclusion is immediate from the induction hypothesis. If $\texttt{U} = \texttt{P}$, then we have $\Delta \vdash \texttt{P} <: \texttt{C<}\overline{\texttt{S}}\texttt{>}$. Then the conclusion is immediate from the induction hypothesis.

Case $\Delta \vdash \texttt{dyn<C<}\overline{\texttt{T}}\texttt{>>} <: \texttt{C<}\overline{\texttt{T}}\texttt{>}$ ($\texttt{N} = \texttt{C<}\overline{\texttt{T}}\texttt{>}$). Immediate from Lemma 17.

□

Lemma 20. If $\Delta \vdash \texttt{dyn<N>} <: \texttt{T}$ and $\texttt{T} \neq \texttt{dyn<N>}$, then $\Delta \vdash \texttt{N} <: \texttt{T}$.

Proof. By induction on the derivation of $\Delta \vdash \texttt{dyn<N>} <: \texttt{T}$.

Case $\dfrac{\Delta \vdash \texttt{dyn<N>} <: \texttt{U} \qquad \Delta \vdash \texttt{U} <: \texttt{T}}{\Delta \vdash \texttt{dyn<N>} <: \texttt{T}}$. By the induction hypothesis, we have $\Delta \vdash \texttt{N} <: \texttt{U}$. Then, $\dfrac{\Delta \vdash \texttt{N} <: \texttt{U} \qquad \Delta \vdash \texttt{U} <: \texttt{T}}{\Delta \vdash \texttt{N} <: \texttt{T}}$ finishes the case.

Case $\Delta \vdash \texttt{dyn<N>} <: \texttt{N}$ ($\texttt{T} = \texttt{N}$). Immediate.

□

Lemma 21. If $\Delta \vdash \texttt{T} \prec \texttt{C<}\overline{\texttt{S}}\texttt{>}$, then $\texttt{T} = \texttt{C<}\overline{\texttt{T}}\texttt{>}$.

Proof. By induction on the derivation of $\Delta \vdash \texttt{T} \prec \texttt{C<}\overline{\texttt{S}}\texttt{>}$.

Case $\Delta \vdash \texttt{C<}\overline{\texttt{S}}\texttt{>} \prec \texttt{C<}\overline{\texttt{S}}\texttt{>}$ ($\texttt{T} = \texttt{C<}\overline{\texttt{S}}\texttt{>}$). Immediate.

Case $\dfrac{\Delta \vdash \texttt{T} \prec \texttt{U} \qquad \Delta \vdash \texttt{U} \prec \texttt{C<}\overline{\texttt{S}}\texttt{>}}{\Delta \vdash \texttt{T} \prec \texttt{C<}\overline{\texttt{S}}\texttt{>}}$. By the induction hypothesis, we have $\texttt{U} = \texttt{C<}\overline{\texttt{U}}\texttt{>}$. Then, the conclusion is immediate from the induction hypothesis.

Case $\dfrac{\Delta \vdash \overline{\texttt{T}} \prec \overline{\texttt{S}}}{\Delta \vdash \texttt{C<}\overline{\texttt{T}}\texttt{>} \prec \texttt{C<}\overline{\texttt{S}}\texttt{>}}$ ($\texttt{T} = \texttt{C<}\overline{\texttt{T}}\texttt{>}$, $\texttt{N} = \texttt{C<}\overline{\texttt{S}}\texttt{>}$). Immediate.

□

Lemma 22. If $\Delta \vdash \mathtt{T} \prec \mathtt{N}$, then $depth_\Delta(\mathtt{T}) = depth_\Delta(\mathtt{N})$.

Proof. Suppose $\mathtt{N} = \mathtt{C<}\overline{\mathtt{S}}\mathtt{>}$. By Lemma 21, we have $\mathtt{T} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$. Then, by Lemma 12, we have $depth_\Delta(\mathtt{T}) = depth_\Delta(\mathtt{N})$. □

Lemma 23. If $\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{T}}$, then $\Delta \vdash [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{T} \prec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{T}$ for any $\mathtt{T}$.

Proof. By the induction on the structure of $\mathtt{T}$.

Case $\mathtt{T} = \mathtt{X}$. Immediate from $\Delta \vdash \mathtt{S}_i \prec \mathtt{T}_i$ when $\mathtt{X} = \mathtt{X}_i$, or from $\Delta \vdash \mathtt{X} \prec \mathtt{X}$ otherwise.

Case $\mathtt{T} = \mathtt{C<}\overline{\mathtt{U}}\mathtt{>}$. $\dfrac{\dfrac{\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{T}}}{\Delta \vdash [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \prec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}}\ \text{IH}}{\Delta \vdash \mathtt{C<}[\overline{\mathtt{S}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}\mathtt{>} \prec \mathtt{C<}[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}\mathtt{>}}$.

Case $\mathtt{T} = \mathtt{dyn<N>}$. $\dfrac{\Delta \vdash \mathtt{dyn<}[\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N}\mathtt{>} <: [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N} \quad \dfrac{\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{T}}}{\Delta \vdash [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N} \prec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}}\ \text{IH}}{\Delta \vdash \mathtt{dyn<}[\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N}\mathtt{>} \prec \mathtt{dyn<}[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}\mathtt{>}}$.

□

Lemma 24. If $\Delta \vdash \mathtt{S} \prec \mathtt{U}$ and $\Delta \vdash \mathtt{U} <: \mathtt{T}$ and $\mathtt{U} \neq \mathtt{T}$, then $\Delta \vdash \mathtt{S} <: \mathtt{V}$ and $\Delta \vdash \mathtt{V} \prec \mathtt{T}$ for some $\mathtt{V}$ such that $depth_\Delta(\mathtt{V}) < depth_\Delta(\mathtt{U})$.

Proof. By induction on the derivation of $\Delta \vdash \mathtt{U} <: \mathtt{T}$.

Case $\dfrac{\Delta \vdash \mathtt{U} <: \mathtt{U}_1 \qquad \Delta \vdash \mathtt{U}_1 <: \mathtt{T}}{\Delta \vdash \mathtt{U} <: \mathtt{T}}$. If $\mathtt{U} = \mathtt{U}_1$ or $\mathtt{T} = \mathtt{U}_1$, then the conclusion is immediate from the induction hypothesis. Otherwise ($\mathtt{U} \neq \mathtt{U}_1$ and $\mathtt{T} \neq \mathtt{U}_1$), by the induction hypothesis, $\Delta \vdash \mathtt{S} <: \mathtt{U}_2$ and $\Delta \vdash \mathtt{U}_2 \prec \mathtt{U}_1$, $\Delta \vdash \mathtt{U}_2 <: \mathtt{V}$ and $\Delta \vdash \mathtt{V} \prec \mathtt{T}$ for some $\mathtt{U}_2$, $\mathtt{V}$ such that $depth_\Delta(\mathtt{U}_2) < depth_\Delta(\mathtt{U})$ and $depth_\Delta(\mathtt{V}) < depth_\Delta(\mathtt{U}_1)$. Then, by Lemma 14, $depth_\Delta(\mathtt{V}) \leq depth_\Delta(\mathtt{U}_2) < depth_\Delta(\mathtt{U})$.

Case $\Delta \vdash \mathtt{X} <: bound_\Delta(\mathtt{X})$ (where $\mathtt{U} = \mathtt{X}$, $\mathtt{T} = bound_\Delta(\mathtt{X})$). Clearly, $\mathtt{S} = \mathtt{X}$ by a rule matches $\Delta \vdash \mathtt{S} \prec \mathtt{X}$. Letting $\mathtt{V} = \mathtt{T}$ and Lemma 13 finishes the case.

Case $\dfrac{\mathtt{class\ C<}\overline{\mathtt{X}}^{\kappa} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \mathtt{\{...\}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{U}}\mathtt{>} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}}$ (where $\mathtt{U} = \mathtt{C<}\overline{\mathtt{U}}\mathtt{>}$, $\mathtt{T} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}$). By $\dfrac{\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{U}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{S}}\mathtt{>} \prec \mathtt{C<}\overline{\mathtt{U}}\mathtt{>}}$ (where $\mathtt{S} = \mathtt{C<}\overline{\mathtt{S}}\mathtt{>}$) and Lemma 23, $\Delta \vdash [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N} \prec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}$. By Lemma 12, we have $depth_\Delta([\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N}) = depth_\Delta([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N})$. $\dfrac{\mathtt{class\ C<}\overline{\mathtt{X}}^{\kappa} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \mathtt{\{...\}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{S}}\mathtt{>} <: [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N}}$ finishes the case with $\mathtt{V} = [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N}$, where $depth_\Delta(\mathtt{V}) = depth_\Delta([\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{N}) = depth_\Delta([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}) < depth_\Delta(\mathtt{C<}\overline{\mathtt{U}}\mathtt{>}) = depth_\Delta(\mathtt{U})$.

Case $\Delta \vdash \mathtt{dyn<N>} <: \mathtt{N}$ (where $\mathtt{U} = \mathtt{dyn<N>}$, $\mathtt{T} = \mathtt{N}$). If $\mathtt{S} = \mathtt{U}$, then the conclusion is immediate from the induction hypothesis. Otherwise, we have $\dfrac{\Delta \vdash \mathtt{S} <: \mathtt{V} \qquad \Delta \vdash \mathtt{V} \prec \mathtt{N}}{\Delta \vdash \mathtt{S} \prec \mathtt{dyn<N>}}$. Then, by Lemma 22 and Lemma 13, we have $depth_\Delta(\mathtt{V}) = depth_\Delta(\mathtt{N}) = depth_\Delta(\mathtt{T}) < depth_\Delta(\mathtt{U})$.

□

Lemma 25. If $\Delta \vdash \mathtt{S} \prec \mathtt{U}$ and $\Delta \vdash \mathtt{U} <: \mathtt{T}$, then $\Delta \vdash \mathtt{S} <: \mathtt{V}$ and $\Delta \vdash \mathtt{V} \prec \mathtt{T}$ for some $\mathtt{V}$.

Proof. If $\mathtt{U} \neq \mathtt{T}$, then the conclusion is immediate from Lemma 24. Otherwise, the conclusion is immediate from letting $\mathtt{V} = \mathtt{S}$. □

Lemma 26. If $\Delta \vdash \mathtt{S} \nprec \mathtt{U}$ and $\Delta \vdash \mathtt{U} \nprec \mathtt{T}$, then $\Delta \vdash \mathtt{S} \nprec \mathtt{T}$.

Proof. Immediate from Lemma 25 and transitivity of $<:$ and $\prec$. □

B.2 Conservative Typing of FGJ$^{\mathrm{dyn}}$

B.2.1 Proof of Lemma 5

Proof.

($\prec$) $\Delta \vdash \mathtt{S} \prec \mathtt{T} \quad \mathit{dynfree}_\Delta(\mathtt{T})$
By induction on the derivation of $\Delta \vdash \mathtt{S} \prec \mathtt{T}$ with a case analysis on the last rule used.
Case $\Delta \vdash \mathtt{T} \prec \mathtt{T}$ (where $\mathtt{S} = \mathtt{T}$). Immediate from $\mathtt{S} = \mathtt{T}$.
Case $\dfrac{\Delta \vdash \mathtt{S} \prec \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{T}}{\Delta \vdash \mathtt{S} \prec \mathtt{T}}$. By the induction hypothesis, we have $\mathtt{U} = \mathtt{T}$ and $\mathit{dynfree}_\Delta(\mathtt{U})$. Then the conclusion is immediate from the induction hypothesis.
Case $\dfrac{\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{T}}}{\Delta \vdash \mathtt{C}\texttt{<}\overline{\mathtt{S}}\texttt{>} \prec \mathtt{C}\texttt{<}\overline{\mathtt{T}}\texttt{>}}$ ($\mathtt{S} = \mathtt{C}\texttt{<}\overline{\mathtt{S}}\texttt{>}$, $\mathtt{T} = \mathtt{C}\texttt{<}\overline{\mathtt{T}}\texttt{>}$). The conclusion is immediate from the induction hypothesis.

$\Delta \vdash \mathtt{S} <: \mathtt{U} \quad \Delta \vdash \mathtt{U} \prec \mathtt{T}$
($\precsim$) By the first statement of this lemma, we have $\mathtt{U} = \mathtt{T}$. Then, we have $\Delta \vdash \mathtt{S} <: \mathtt{T}$.
($\precsim$-1) $\Delta \vdash \mathtt{U} \prec \mathtt{S} \quad \Delta \vdash \mathtt{U} <: \mathtt{V} \quad \Delta \vdash \mathtt{V} \prec \mathtt{T}$
By the first statement of this lemma, we have $\mathtt{U} = \mathtt{S}$. Then, we have $\Delta \vdash \mathtt{S} \precsim \mathtt{T}$.
($\precsim$-2) Immediate from the second and the third statement of this lemma.

□

B.2.2 Proof of Theorem 6

Proof. Since `dyn<P>` does not appear in $(CT, \mathtt{e})$, bound and $\mathit{override}$ in FGJ$^{\mathrm{dyn}}$ are equivalent to those in FGJ, and premises of TG-FIELD2 and TG-INVK2 will never satisfied, and by Lemma 5, TG-NEW, TG-FIELD1, TG-INVK1 and TG-METHOD are equivalent to corresponding rules in FGJ. So, typing rules of FGJ$^{\mathrm{dyn}}$ are equivalent to FGJ's. □

B.3 FGJ$^{(\!(\,)\!)}$ type safety

We use $\Delta_1, \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}}, \Delta_2 \vdash [\overline{\mathtt{T}}/\overline{\mathtt{X}}]$ ok to state that $\mathit{kind}_{\Delta_1, \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}}, \Delta_2}(\mathtt{X}_i) = \Diamond$ implies $\mathit{dynfree}_{\Delta_1, \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}}, \Delta_2}(\mathtt{T}_i)$ for any i.

Lemma 27. Suppose $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \overline{\mathtt{N}}$ ok and $\Delta \vdash \mathtt{U}$ ok.

1. If $\Delta \vdash \mathtt{S} <: \mathtt{T}$, then $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{S} <: \mathtt{T}$.
2. If $\Delta \vdash \mathtt{S} \prec \mathtt{T}$, then $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{S} \prec \mathtt{T}$.
3. If $\Delta \vdash \mathtt{S}$ ok, then $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{S}$ ok.
4. If $\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{T}$, then $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{T}$.

Proof.

1. By induction on the derivation of $\Delta \vdash \mathtt{S} <: \mathtt{T}$ with a case analysis on the last rule used.
Case $\Delta \vdash \mathtt{S} <: \mathtt{S}$ ($\mathtt{T} = \mathtt{S}$). Immediate from $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{S} <: \mathtt{S}$.
Case $\dfrac{\Delta \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta \vdash \mathtt{U} <: \mathtt{T}}{\Delta \vdash \mathtt{S} <: \mathtt{T}}$. Immediate from the induction hypothesis.
Case $\Delta \vdash \mathtt{X} <: \mathit{bound}_\Delta(\mathtt{X})$ ($\mathtt{S} = \mathtt{X}$, $\mathtt{T} = \mathit{bound}_\Delta(\mathtt{X})$). Immediate from $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{X} <: \mathit{bound}_{\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}}(\mathtt{X})$.
Case $\dfrac{\mathtt{class\ C}\texttt{<}\overline{\mathtt{Y}}^{\overline{\kappa}} \lhd \overline{\mathtt{P}}\texttt{>} \lhd \mathtt{N}\,\{\ldots\}}{\Delta \vdash \mathtt{C}\texttt{<}\overline{\mathtt{T}}\texttt{>} <: [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{N}}$ ($\mathtt{S} = \mathtt{C}\texttt{<}\overline{\mathtt{T}}\texttt{>}$, $\mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{N}$). Immediate from $\dfrac{\mathtt{class\ C}\texttt{<}\overline{\mathtt{Y}}^{\overline{\kappa}} \lhd \overline{\mathtt{P}}\texttt{>} \lhd \mathtt{N}\,\{\ldots\}}{\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{C}\texttt{<}\overline{\mathtt{T}}\texttt{>} <: [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{N}}$.
Case $\Delta \vdash \mathtt{dyn}\texttt{<}\mathtt{N}\texttt{>} <: \mathtt{N}$ ($\mathtt{S} = \mathtt{dyn}\texttt{<}\mathtt{N}\texttt{>}$, $\mathtt{T} = \mathtt{N}$). Immediate from $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{dyn}\texttt{<}\mathtt{N}\texttt{>} <: \mathtt{N}$.
2. By induction on the derivation of $\Delta \vdash \mathtt{S} \prec \mathtt{T}$ with a case analysis on the last rule used.
Case $\Delta \vdash \mathtt{S} \prec \mathtt{S}$ ($\mathtt{T} = \mathtt{S}$). Immediate from $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{S} \prec \mathtt{S}$.
Case $\dfrac{\Delta \vdash \mathtt{S} \prec \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{T}}{\Delta \vdash \mathtt{S} \prec \mathtt{T}}$. Immediate from the induction hypothesis.
Case $\dfrac{\Delta \vdash \overline{\mathtt{U}} \prec \overline{\mathtt{V}}}{\Delta \vdash \mathtt{C}\texttt{<}\overline{\mathtt{U}}\texttt{>} \prec \mathtt{C}\texttt{<}\overline{\mathtt{V}}\texttt{>}}$ ($\mathtt{S} = \mathtt{C}\texttt{<}\overline{\mathtt{U}}\texttt{>}$, $\mathtt{T} = \mathtt{C}\texttt{<}\overline{\mathtt{V}}\texttt{>}$). Immediate from the induction hypothesis.

Case $\dfrac{\Delta \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{N}}{\Delta \vdash \mathtt{S} \prec \mathtt{dyn\texttt{<}N\texttt{>}}}$ ($\mathtt{T} = \mathtt{dyn\texttt{<}N\texttt{>}}$). Immediate from the induction hypothesis.

3. By induction on the derivation of $\Delta \vdash \mathtt{S}$ ok with a case analysis on the last rule used.

Case $\Delta \vdash \mathtt{Object}$ ok ($\mathtt{S} = \mathtt{Object}$). Immediate from $\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{Object}$ ok.

Case $\dfrac{\mathtt{X} \in \mathit{dom}(\Delta)}{\Delta \vdash \mathtt{X}\ \mathrm{ok}}$ ($\mathtt{S} = \mathtt{X}$). Immediate from $\dfrac{\mathtt{X} \in \mathit{dom}(\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}})}{\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{X}\ \mathrm{ok}}$.

Case $\dfrac{\begin{array}{c}\mathtt{class\ C\texttt{<}}\overline{\mathtt{Y}}^{\overline{\kappa}} \lhd \overline{\mathtt{P}}\mathtt{\texttt{>}} \lhd \mathtt{N}\ \mathtt{\{\ldots\}} \qquad \Delta \vdash \overline{\mathtt{T}}\ \mathrm{ok} \\ \forall \kappa_i \in \overline{\kappa}.(\ \kappa_i = \Diamond \text{ implies } \mathit{dynfree}_{\Delta}(\mathtt{T}_i)\ ) \\ \forall \mathtt{T}_i \in \overline{\mathtt{T}}.(\ \Delta \vdash \mathtt{T}_i \nprec [\,\mathtt{T}_1/\mathtt{Y}_1, \ldots, \mathtt{T}_{i-1}/\mathtt{Y}_{i-1}\,]\mathtt{P}_i\ )\end{array}}{\Delta \vdash \mathtt{C\texttt{<}}\overline{\mathtt{T}}\mathtt{\texttt{>}}\ \mathrm{ok}}$ ($\mathtt{S} = \mathtt{C\texttt{<}}\overline{\mathtt{T}}\mathtt{\texttt{>}}$). Immediate from the induction hypothesis and

$\dfrac{\begin{array}{c}\mathtt{class\ C\texttt{<}}\overline{\mathtt{Y}}^{\overline{\kappa}} \lhd \overline{\mathtt{P}}\mathtt{\texttt{>}} \lhd \mathtt{N}\ \mathtt{\{\ldots\}} \qquad \Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \overline{\mathtt{T}}\ \mathrm{ok} \\ \forall \kappa_i \in \overline{\kappa}.(\ \kappa_i = \Diamond \text{ implies } \mathit{dynfree}_{\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}}(\mathtt{T}_i)\ ) \\ \forall \mathtt{T}_i \in \overline{\mathtt{T}}.(\ \Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{T}_i \nprec [\,\mathtt{T}_1/\mathtt{Y}_1, \ldots, \mathtt{T}_{i-1}/\mathtt{Y}_{i-1}\,]\mathtt{P}_i\ )\end{array}}{\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{C\texttt{<}}\overline{\mathtt{T}}\mathtt{\texttt{>}}\ \mathrm{ok}}$.

Case $\dfrac{\Delta \vdash \mathtt{N}\ \mathrm{ok}}{\Delta \vdash \mathtt{dyn\texttt{<}N\texttt{>}}\ \mathrm{ok}}$ ($\mathtt{S} = \mathtt{dyn\texttt{<}N\texttt{>}}$). Immediate from the induction hypothesis.

4. By induction on the derivation of $\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{T}$ with a case analysis on the last rule used.

Case Tr-Var.
$\mathtt{e} = \mathtt{x} \quad \mathtt{T} = \Gamma(\mathtt{x})$
Immediate from the rule Tr-Var.

Case Tr-Field1.
$\mathtt{e} = \mathtt{e}_0\mathtt{.f}_i \quad \Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\mathit{fields}(\mathit{bound}_{\Delta}(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \quad \mathtt{T} = \mathtt{T}_i$
By the induction hypothesis, we have
$\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
Then, applying the rule Tr-Field1 finishes the case.

Case Tr-Field2.
$\mathtt{e} = \mathtt{get(e}_0\mathtt{,\ f)} \quad \Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \mathtt{T} = \mathtt{dyn\texttt{<}Object\texttt{>}}$
By the induction hypothesis, we have
$\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
Then, applying the rule Tr-Field2 finishes the case.

Case Tr-Invk1.
$\mathtt{e} = \mathtt{e}_0\mathtt{.m(}\overline{\mathtt{e}}\mathtt{)} \quad \Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathit{bound}_{\Delta}(\mathtt{T}_0) = \mathtt{P} \quad \Delta \vdash \mathtt{P} \nprec \mathtt{N}$
$\mathit{dynfree}_{\Delta}(\mathtt{N}) \quad \mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{T} \quad \Delta \vdash \overline{\mathtt{V}} \nprec \overline{\mathtt{S}}$
By the induction hypothesis, we have
$\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$
By the first and second statement of this lemma, we have
$\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \overline{\mathtt{V}} \nprec \overline{\mathtt{S}} \quad \Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{P} \nprec \mathtt{N}$
Then, applying the rule Tr-Invk1 finishes the case.

Case Tr-Invk2.
$\mathtt{e} = \mathtt{e}_0\mathtt{.m[}\overline{\mathtt{U}}\mathtt{|C\texttt{<}}\overline{\mathtt{Y}}\mathtt{\texttt{>](}}\overline{\mathtt{e}}\mathtt{)} \quad \Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathit{bound}_{\Delta}(\mathtt{T}_0) = \mathtt{N} \quad \Delta \vdash \mathtt{N} \nprec [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{C\texttt{<}}\overline{\mathtt{Y}}\mathtt{\texttt{>}}$
$\mathit{mtype}(\mathtt{m}, \mathtt{C\texttt{<}}\overline{\mathtt{Y}}\mathtt{\texttt{>}}) = \overline{\mathtt{U}} \to \mathtt{U} \quad \Delta \vdash \overline{\mathtt{V}} \nprec [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}} \quad \mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{U}$
By the induction hypothesis, we have
$\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$
By the first and second statement of this lemma, we have

$\Delta, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{N} \not\asymp [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{C<}\overline{\mathtt{Y}}\mathtt{>}$
$\Delta, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}} \vdash \overline{\mathtt{V}} \not\asymp [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}}$
Then, applying the rule TR-INVK2 finishes the case.

Case TR-INVK3.
$\mathtt{e} = \mathtt{invoke(e}_0\mathtt{,m,}\overline{\mathtt{e}}\mathtt{)} \quad \Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathtt{T} = \mathtt{dyn<Object>}$
By the induction hypothesis, we have
$\Delta, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$
Then, applying the rule TR-INVK3 finishes the case.

Case TR-NEW.
$\mathtt{e} = \mathtt{new\ N(}\overline{\mathtt{e}}\mathtt{)} \quad \Delta \vdash \mathtt{N}$ ok
$\mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \quad \Delta; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$
$\Delta \vdash \overline{\mathtt{V}} \not\asymp \overline{\mathtt{T}} \quad \mathtt{T} = \mathtt{N}$
By the induction hypothesis, we have
$\Delta, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$
By the first and second statement of this lemma, we have
$\Delta, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}} \vdash \overline{\mathtt{V}} \not\asymp \overline{\mathtt{T}}$
By the third statement of this lemma, we have
$\Delta, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}} \vdash \mathtt{N}$ ok
Then, applying the rule TR-NEW finishes the case.

Case TR-CAST.
$\mathtt{e} = (\!|\mathtt{T}|\!)\,\mathtt{e}_0 \quad \Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}$
By the induction hypothesis, we have
$\Delta, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}$
Then, applying the rule TR-CAST finishes the case.

Case TR-ERROR.
$\mathtt{e} = \mathtt{Error}[\mathcal{E}]$
Immediate from the rule TR-ERROR.

□

Lemma 28. If $\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{S} <: \mathtt{T}$ where $\mathtt{S} \neq \mathtt{X}_i$ and none of $\overline{\mathtt{X}}$ appears in Δ_1, then $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{S} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$.

Proof. By induction on the derivation of $\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{S} <: \mathtt{T}$.

Case $\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{S} <: \mathtt{S}$ ($\mathtt{T} = \mathtt{S}$). Immediate from $[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{S} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$.

Case $\dfrac{\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{U} <: \mathtt{T}}{\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{S} <: \mathtt{T}}$. We have $\mathtt{U} \neq \mathtt{X}_i$ because if $\mathtt{U} = \mathtt{X}_i$, then $\mathtt{S} = \mathtt{X}_i$ by Lemma 15, which is a contradiction. By the induction hypothesis, we have
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{S} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{U} \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{U} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$
Then, $\dfrac{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{S} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{U} \qquad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{U} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}}{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{S} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}}$ finishes the case.

Case $\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{X} <: \mathit{bound}_{\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2}(\mathtt{X})$ ($\mathtt{S} = \mathtt{X}$, $\mathtt{T} = \mathit{bound}_{\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2}(\mathtt{X})$). By the assumption, we have
$\mathit{bound}_{\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2}(\mathtt{X}) = \mathit{bound}_{\Delta_1, \Delta_2}(\mathtt{X})$
$[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathit{bound}_{\Delta_1, \Delta_2}(\mathtt{X}) = \mathit{bound}_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{X})$
$[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{S} = \mathtt{X} \quad [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T} = \mathit{bound}_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{X})$
Then, $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{X} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathit{bound}_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{X})$ finishes the case.

Case $\dfrac{\mathtt{class\ C<}\overline{\mathtt{Y}^{\kappa}} \mathtt{\lhd}\ \overline{\mathtt{P}}\mathtt{>} \lhd \mathtt{N\ \{...\}}}{\Delta_1, \overline{\mathtt{X}^{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{C<}\overline{\mathtt{T}}\mathtt{>} <: [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{N}}$ ($\mathtt{S} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$, $\mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{N}$). By TR-CLASS, none of $\overline{\mathtt{X}}$ appears in $\mathtt{N}$ and we have $[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N} = \mathtt{N}$.

Then, $\dfrac{\texttt{class C<}\overline{\texttt{Y}}^{\overline{\kappa}}\texttt{<:}\overline{\texttt{P}}\texttt{> <: N \{...\}}}{\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{C<}[\overline{\texttt{U}}/\overline{\texttt{X}}]\overline{\texttt{T}}\texttt{> <: } [\,[\overline{\texttt{U}}/\overline{\texttt{X}}]\overline{\texttt{T}}/\overline{\texttt{Y}}\,]\texttt{N}}$ finishes the case.

Case $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{dyn<N> <: N}$ ($\texttt{S} = \texttt{dyn<N>}, \texttt{T} = \texttt{N}$).
Immediate from $\Delta_1, [\overline{\texttt{X}}/\overline{\texttt{N}}]\Delta_2 \vdash \texttt{dyn<}[\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N> <: } [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}$.

□

Lemma 29. If $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S <: T}$ and $\Delta_1 \vdash \overline{\texttt{U}} \nprec [\overline{\texttt{U}}/\overline{\texttt{X}}]\overline{\texttt{N}}$ where $\Delta_1 \vdash \overline{\texttt{U}}$ ok and none of $\overline{\texttt{X}}$ appears in Δ_1, then $\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{S} \nprec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{T}$.

Proof. If $\texttt{S} \neq \texttt{X}_i$, then the conclusion is immediate from Lemma 28. Otherwise, by induction on the derivation of $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S <: T}$.

Case $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{X}_i \texttt{<: X}_i$ ($\texttt{T} = \texttt{S} = \texttt{X}_i$). We have $[\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{X}_i = \texttt{U}_i$ and the conclusion is immediate from $\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{U}_i \nprec \texttt{U}_i$.

Case $\dfrac{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{X}_i \texttt{<: U} \qquad \Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{U <: T}}{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{X}_i \texttt{<: T}}$. The conclusion is immediate from the induction hypothesis and Lemma 26.

Case $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{X}_i \texttt{<: } \mathit{bound}_{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2}(\texttt{X}_i)$ ($\texttt{T} = \mathit{bound}_{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2}(\texttt{X}_i)$). We have $[\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{S} = \texttt{U}_i$ and $\mathit{bound}_{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2}(\texttt{X}_i) = \texttt{N}_i = \texttt{T}$. Then, we have $\Delta_1 \vdash \texttt{U}_i \nprec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}_i$ by the assumption and the conclusion is immediate from Lemma 27.

□

Lemma 30. If $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S} \prec \texttt{T}$ and $\Delta_1 \vdash \overline{\texttt{U}} \nprec [\overline{\texttt{U}}/\overline{\texttt{X}}]\overline{\texttt{N}}$ where $\Delta_1 \vdash \overline{\texttt{U}}$ ok and none of $\overline{\texttt{X}}$ appears in Δ_1, then $\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{S} \prec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{T}$.

Proof. By induction on the derivation of $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S} \prec \texttt{T}$.

Case $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S} \prec \texttt{S}$ ($\texttt{T} = \texttt{S}$). Immediate.

Case $\dfrac{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S} \prec \texttt{U} \qquad \Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{U} \prec \texttt{T}}{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S} \prec \texttt{T}}$. Immediate from the induction hypothesis.

Case $\dfrac{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \overline{\texttt{S}} \prec \overline{\texttt{T}}}{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{C<}\overline{\texttt{S}}\texttt{>} \prec \texttt{C<}\overline{\texttt{T}}\texttt{>}}$ ($\texttt{S} = \texttt{C<}\overline{\texttt{S}}\texttt{>}, \texttt{T} = \texttt{C<}\overline{\texttt{T}}\texttt{>}$). Immediate from the induction hypothesis.

Case $\dfrac{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S <: U} \qquad \Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{U} \prec \texttt{N}}{\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S} \prec \texttt{dyn<N>}}$ ($\texttt{T} = \texttt{dyn<N>}$). If $\texttt{S} \neq \texttt{X}_i$, then the conclusion is immediate from Lemma 28 and the induction hypothesis. Otherwise, we have

$[\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{S} = \texttt{U}_i \quad \texttt{U} = \texttt{N}_i$

By Lemma 29 and the induction hypothesis, we have

$\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{U}_i \texttt{<: V}$

$\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{V} \prec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}_i$

$\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}_i \prec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}$

for some V. Then, $\dfrac{\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{U}_i \texttt{<: V} \quad \Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{V} \prec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}}{\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{U}_i \prec \texttt{dyn<}[\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N>}}$,

$\dfrac{\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{V} \prec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}_i \quad \Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}_i \prec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}}{\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash \texttt{V} \prec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{N}}$

finishes the case.

□

Lemma 31. If $\Delta_1, \overline{\texttt{X}}^{\overline{\iota}} \texttt{<: } \overline{\texttt{N}}, \Delta_2 \vdash \texttt{S} \nprec \texttt{T}$ and $\Delta_1 \vdash \overline{\texttt{U}} \nprec [\overline{\texttt{U}}/\overline{\texttt{X}}]\overline{\texttt{N}}$ where $\Delta_1 \vdash \overline{\texttt{U}}$ ok and none of $\overline{\texttt{X}}$ appears in Δ_1, then $\Delta_1, [\overline{\texttt{U}}/\overline{\texttt{X}}]\Delta_2 \vdash [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{S} \nprec [\overline{\texttt{U}}/\overline{\texttt{X}}]\texttt{T}$.

Proof. We have

$\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{S} <: \mathtt{U} \quad \Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{U} \prec \mathtt{T}$

for some $\mathtt{U}$. Then the conclusion is immediate from Lemma 29, Lemma 30 and transitivity of $\prec$. □

Lemma 32. Under CT OK IN FGJ$^{()}$, if $dynfree_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} \lhd \overline{\mathtt{N}}, \Delta_2}(\mathtt{T})$ and $\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]$ ok where $\Delta_1 \vdash \overline{\mathtt{U}}$ ok and none of $\overline{\mathtt{X}}$ appears in Δ_1, then $dynfree_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T})$.

Proof. By induction on the derivation of $dynfree_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} \lhd \overline{\mathtt{N}}, \Delta_2}(\mathtt{T})$.

Case $\dfrac{kind_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2}(\mathtt{Y}) = \Diamond}{dynfree_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} \lhd \overline{\mathtt{N}}, \Delta_2}(\mathtt{Y})}$ ($\mathtt{T} = \mathtt{Y}$). If $\mathtt{Y} = \mathtt{X}_i$ for some i, then we have $kind_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2}(\mathtt{X}) = \Diamond$ and $dynfree_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} \lhd \overline{\mathtt{N}}, \Delta_2}(\mathtt{U}_i)$. Since $\Delta_1 \vdash \overline{\mathtt{U}}$ ok, we have $dynfree_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{U}_i)$ where $[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T} = \mathtt{U}_i$. Otherwise, the conclusion is immediate since $[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T} = \mathtt{T}$.

Case $\dfrac{dynfree_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} \lhd \overline{\mathtt{N}}, \Delta_2}(\overline{\mathtt{T}})}{dynfree_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} \lhd \overline{\mathtt{N}}, \Delta_2}(\mathtt{C<}\overline{\mathtt{T}}\mathtt{>})}$ ($\mathtt{T} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$). The conclusion is immediate from the induction hypothesis.

□

Lemma 33. Under CT OK IN FGJ$^{()}$, if $\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{T}$ ok and $\Delta_1 \vdash \overline{\mathtt{U}} \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$ where $\Delta_1 \vdash \overline{\mathtt{U}}$ ok and none of $\overline{\mathtt{X}}$ appears in Δ_1 and $\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]$ ok, then $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$ ok.

Proof. By induction on the derivation of $\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{T}$ ok with a case analysis on the last rule used.

Case $\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{Object}$ ok ($\mathtt{T} = \mathtt{Object}$). Immediate from $[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{Object} = \mathtt{Object}$.

Case $\dfrac{\mathtt{X} \in dom(\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2)}{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{X}\ \mathrm{ok}}$ ($\mathtt{T} = \mathtt{X}$). If $\mathtt{X} = \mathtt{X}_i$, then we have $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{U}_i$ ok by the assumption and Lemma 27. Otherwise, immediate from $[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{X} = \mathtt{X}$.

Case $\dfrac{\begin{array}{c}\mathtt{class\ C<}\overline{\mathtt{Y}}^{\overline{\kappa}} \lhd \overline{\mathtt{P}}\mathtt{>} \lhd \mathtt{N}\ \mathtt{\{...\}} \qquad \Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{T}}\ \mathrm{ok} \\ \forall \kappa_i \in \overline{\kappa}.(\ \kappa_i = \Diamond \text{ implies } dynfree_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} \lhd \overline{\mathtt{N}}, \Delta_2}(\mathtt{T}_i)\ ) \\ \forall \mathtt{T}_i \in \overline{\mathtt{T}}.(\ \Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{T}_i \nprec [\mathtt{T}_1/\mathtt{Y}_1, \ldots, \mathtt{T}_{i-1}/\mathtt{Y}_{i-1}]\mathtt{P}_i\ )\end{array}}{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}\ \mathrm{ok}}$ ($\mathtt{T} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$). By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}$ ok

By Lemma 31, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_i \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}][\mathtt{T}_1/\mathtt{Y}_1, \ldots, \mathtt{T}_{i-1}/\mathtt{Y}_{i-1}]\mathtt{P}_i$

Since $\mathtt{Y}_1^{\kappa_1} <: \mathtt{P}_1, \ldots, \mathtt{Y}_{i-1}^{\kappa_{i-1}} <: \mathtt{P}_{i-1} \vdash \mathtt{P}_i$ ok by the rule TR-CLASS, $\overline{\mathtt{P}}$ does not include any $\overline{\mathtt{X}}$ as a free variable. Thus, $[\overline{\mathtt{U}}/\overline{\mathtt{X}}][\mathtt{T}_1/\mathtt{Y}_1, \ldots, \mathtt{T}_{i-1}/\mathtt{Y}_{i-1}]\mathtt{P}_i = [\,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_1/\mathtt{Y}_1, \ldots, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_{i-1}/\mathtt{Y}_{i-1}]\mathtt{P}_i$. Suppose $\kappa_i = \Diamond$ for some i. We have $dynfree_{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} \lhd \overline{\mathtt{N}}, \Delta_2}(\mathtt{T}_i)$. Then, by Lemma 32, we have $dynfree_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T})$, and finally,

$$\frac{\begin{array}{c}\mathtt{class\ C<}\overline{\mathtt{Y}}^{\overline{\kappa}} \lhd \overline{\mathtt{P}}\mathtt{>} \lhd \mathtt{N}\ \mathtt{\{...\}} \qquad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\ \mathrm{ok} \\ \forall \kappa_i \in \overline{\kappa}.(\ \kappa_i = \Diamond \text{ implies } dynfree_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_i)\ ) \\ \forall [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_i \in [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}.(\ \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_i \nprec [\,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_1/\mathtt{Y}_1, \ldots, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_{i-1}/\mathtt{Y}_{i-1}]\mathtt{P}_i\ )\end{array}}{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{C<}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}\mathtt{>}\ \mathrm{ok}}$$ finishes the case.

Case $\dfrac{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{N}\ \mathrm{ok}}{\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{dyn<N>}\ \mathrm{ok}}$ ($\mathtt{T} = \mathtt{dyn<N>}$). By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}$ ok

Then, $\dfrac{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}\ \mathrm{ok}}{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{dyn<}[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}\mathtt{>}\ \mathrm{ok}}$ finishes the case.

□

Lemma 34. If $\Delta_1, \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{T}$ ok and $\Delta_1 \vdash \overline{\mathtt{U}} \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$ where $\Delta_1 \vdash \overline{\mathtt{U}}$ ok and none of $\overline{\mathtt{X}}$ appears in Δ_1, then $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash bound_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}) \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}](bound_{\Delta_1, \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}}, \Delta_2}(\mathtt{T}))$.

Proof. If $\mathtt{T} \neq \mathtt{X}$, then the conclusion is immediate from $bound_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}) = [\overline{\mathtt{U}}/\overline{\mathtt{X}}](bound_{\Delta_1,\overline{\mathtt{X}^{\kappa}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T}))$. Otherwise, if $\mathtt{T} = \mathtt{X}$ and $\mathtt{X} \in dom(\Delta_1) \cup dom(\Delta_2)$, then the conclusion is immediate from $bound_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}) = [\overline{\mathtt{U}}/\overline{\mathtt{X}}](bound_{\Delta_1,\overline{\mathtt{X}^{\kappa}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T}))$. Finally, if $\mathtt{T} = \mathtt{X}_i$, then we have

$bound_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}) = bound_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{U}_i)$

$[\overline{\mathtt{U}}/\overline{\mathtt{X}}](bound_{\Delta_1,\overline{\mathtt{X}^{\kappa}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T})) = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}_i$

If $\mathtt{U}_i = \mathtt{Y}$, then we have $\mathtt{Y} \in dom(\Delta_1)$ by the assumption $\Delta_1 \vdash \overline{\mathtt{U}}$ ok. By the assumption $\dfrac{\Delta_1 \vdash \overline{\mathtt{U}} <: \overline{\mathtt{S}} \qquad \Delta_1 \vdash \overline{\mathtt{S}} \prec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}}{\Delta_1 \vdash \overline{\mathtt{U}} \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}}$ (where $\mathtt{S}_i$ must be a nonvariable type by the definition of $\prec$) and $\Delta_1 \vdash \mathtt{Y} <: bound_{\Delta_1}(\mathtt{Y})$, we have

$bound_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{U}_i) = bound_{\Delta_1}(\mathtt{Y}) = \mathtt{S}_i$

Then, $\dfrac{\Delta_1 \vdash \mathtt{S}_i <: \mathtt{S}_i \qquad \Delta_1 \vdash \mathtt{S}_i \prec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}_i}{\Delta_1 \vdash \mathtt{S}_i \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}_i}$ and Lemma 27 finishes the proof. If $\mathtt{U}_i = \mathtt{dyn<N>}$, then we have

$bound_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{U}_i) = \mathtt{N}$

By $\dfrac{\Delta_1 \vdash \mathtt{N} <: \mathtt{N} \qquad \Delta_1 \vdash \mathtt{N} \prec \mathtt{N}}{\Delta_1 \vdash \mathtt{N} \prec \mathtt{dyn<N>}}$, we have

$\Delta_1 \vdash \mathtt{N} \nprec \mathtt{dyn<N>}$

By the assumption $\Delta_1 \vdash \overline{\mathtt{U}} \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$ and Lemma 26, we have

$\Delta_1 \vdash \mathtt{N} \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}_i$

Then, Lemma 27 finishes the proof. If $\mathtt{U}_i = \mathtt{N}$, then we have

$bound_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{U}_i) = \mathtt{U}_i$

Then the assumption $\Delta_1 \vdash \overline{\mathtt{U}} \nprec [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$ and Lemma 27 finishes the proof. □

Lemma 35. If $\Delta \vdash \mathtt{S} <: \mathtt{T}$, then $\Delta \vdash bound_{\Delta}(\mathtt{S}) <: bound_{\Delta}(\mathtt{T})$.

Proof. By induction on the derivation of $\Delta \vdash \mathtt{S} <: \mathtt{T}$ with case analysis on the last rule used.

Case $\Delta \vdash \mathtt{T} <: \mathtt{T}$ ($\mathtt{S} = \mathtt{T}$). Immediate from $bound_{\Delta}(\mathtt{S}) = bound_{\Delta}(\mathtt{T})$

Case $\dfrac{\Delta \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta \vdash \mathtt{U} <: \mathtt{T}}{\Delta \vdash \mathtt{S} <: \mathtt{T}}$. Immediate from the induction hypothesis.

Case $\Delta \vdash \mathtt{X} <: bound_{\Delta}(\mathtt{X})$ ($\mathtt{S} = \mathtt{X}$, $\mathtt{T} = bound_{\Delta}(\mathtt{X})$. Immediate from $bound_{\Delta}(\mathtt{S}) = bound_{\Delta}(\mathtt{T}) = bound_{\Delta}(\mathtt{X})$

Case $\dfrac{\mathtt{class\ C<}\overline{\mathtt{X}^{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N\ \{...\}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{T}}\mathtt{>} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}}$ ($\mathtt{S} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$, $\mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}$). Immediate from $bound_{\Delta}(\mathtt{S}) = \mathtt{S}$ and $bound_{\Delta}(\mathtt{T}) = \mathtt{T}$.

Case $\Delta \vdash \mathtt{dyn<N>} <: \mathtt{N}$ ($\mathtt{S} = \mathtt{dyn<N>}$, $\mathtt{T} = \mathtt{N}$). Immediate from $bound_{\Delta}(\mathtt{S}) = bound_{\Delta}(\mathtt{T}) = \mathtt{N}$

□

Lemma 36. If $\Delta \vdash \mathtt{S} \prec \mathtt{T}$, then $\Delta \vdash bound_{\Delta}(\mathtt{S}) \nprec bound_{\Delta}(\mathtt{T})$.

Proof. By induction on the derivation of $\Delta \vdash \mathtt{S} \prec \mathtt{T}$ with case analysis on the last rule used.

Case $\Delta \vdash \mathtt{T} \prec \mathtt{T}$ ($\mathtt{S} = \mathtt{T}$). Immediate from $bound_{\Delta}(\mathtt{S}) = bound_{\Delta}(\mathtt{T})$

Case $\dfrac{\Delta \vdash \mathtt{S} \prec \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{T}}{\Delta \vdash \mathtt{S} \prec \mathtt{T}}$. Immediate from the induction hypothesis and Lemma 26.

Case $\dfrac{\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{T}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{S}}\mathtt{>} \prec \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}}$ ($\mathtt{S} = \mathtt{C<}\overline{\mathtt{S}}\mathtt{>}$, $\mathtt{T} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$). Immediate from $bound_{\Delta}(\mathtt{S}) = bound_{\Delta}(\mathtt{T})$

Case $\dfrac{\Delta \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{N}}{\Delta \vdash \mathtt{S} \prec \mathtt{dyn<N>}}$ ($\mathtt{T} = \mathtt{dyn<N>}$). By Lemma 35 and the induction hypothesis, we have

$\Delta \vdash bound_{\Delta}(\mathtt{S}) <: bound_{\Delta}(\mathtt{U})$

$\Delta \vdash bound_{\Delta}(\mathtt{U}) \nprec bound_{\Delta}(\mathtt{T})$

Then, Lemma 26 finishes the case.

□

Lemma 37. If $\Delta \vdash \mathtt{S} \nprec \mathtt{T}$, then $\Delta \vdash \mathit{bound}_\Delta(\mathtt{S}) \nprec \mathit{bound}_\Delta(\mathtt{T})$.

Proof. Immediate from Lemma 35, and Lemma 26. □

Lemma 38. If $\Delta \vdash \mathtt{S} <: \mathtt{T}$ and $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T})) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$, then $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{S})) = \overline{\mathtt{S}}\ \overline{\mathtt{g}}$ and $\Delta \vdash \mathtt{S}_i = \mathtt{T}_i$ and $\mathtt{g}_i = \mathtt{f}_i$ for all $i \leq \#(\overline{\mathtt{f}})$.

Proof. By induction on the derivation of $\Delta \vdash \mathtt{S} <: \mathtt{T}$ with case analysis on the last rule used.

Case $\Delta \vdash \mathtt{S} <: \mathtt{S}$ ($\mathtt{T} = \mathtt{S}$). Immediate from $\mathit{bound}_\Delta(\mathtt{S}) = \mathit{bound}_\Delta(\mathtt{T})$.

Case $\dfrac{\Delta \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta \vdash \mathtt{U} <: \mathtt{T}}{\Delta \vdash \mathtt{S} <: \mathtt{T}}$. Immediate from the induction hypothesis and the transitivity of $<:$.

Case $\Delta \vdash \mathtt{X} <: \mathit{bound}_\Delta(\mathtt{X})$ ($\mathtt{S} = \mathtt{X}$, $\mathtt{T} = \mathit{bound}_\Delta(\mathtt{X})$). Immediate from $\mathit{bound}_\Delta(\mathtt{S}) = \mathit{bound}_\Delta(\mathtt{T}) = \mathit{bound}_\Delta(\mathtt{X})$.

Case $\dfrac{\mathtt{class\ C\langle}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{\rangle} \lhd \mathtt{N}\ \mathtt{\{...\}}}{\Delta \vdash \mathtt{C\langle}\overline{\mathtt{T}}\mathtt{\rangle} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}}$ ($\mathtt{S} = \mathtt{C\langle}\overline{\mathtt{T}}\mathtt{\rangle}$, $\mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N}$). By the definition of fields, we have $\mathit{fields}(\mathtt{C\langle}\overline{\mathtt{T}}\mathtt{\rangle}) = \overline{\mathtt{U}}\ \overline{\mathtt{f}}$, $[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}\ \overline{\mathtt{g}}$ where $\overline{\mathtt{U}}\ \overline{\mathtt{f}} = \mathit{fields}([\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{N})$.

Case $\Delta \vdash \mathtt{dyn\langle N\rangle} <: \mathtt{N}$ ($\mathtt{S} = \mathtt{dyn\langle N\rangle}$, $\mathtt{T} = \mathtt{N}$). Immediate from $\mathit{bound}_\Delta(\mathtt{S}) = \mathit{bound}_\Delta(\mathtt{T}) = \mathtt{N}$.

□

Lemma 39. If $\Delta \vdash \mathtt{S} \prec \mathtt{T}$ and $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T})) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$, then $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{S})) = \overline{\mathtt{S}}\ \overline{\mathtt{g}}$ and $\Delta \vdash \mathtt{S}_i \prec \mathtt{T}_i$ and $\mathtt{g}_i = \mathtt{f}_i$ for all $i \leq \#(\overline{\mathtt{f}})$.

Proof. By induction on the derivation of $\Delta \vdash \mathtt{S} \prec \mathtt{T}$ with case analysis on the last rule used.

Case $\Delta \vdash \mathtt{S} \prec \mathtt{S}$ ($\mathtt{T} = \mathtt{S}$). Immediate from $\mathit{bound}_\Delta(\mathtt{S}) = \mathit{bound}_\Delta(\mathtt{T})$.

Case $\dfrac{\Delta \vdash \mathtt{S} \prec \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{T}}{\Delta \vdash \mathtt{S} \prec \mathtt{T}}$. Immediate from the induction hypothesis and the transitivity of $\prec$.

Case $\dfrac{\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{T}}}{\Delta \vdash \mathtt{C\langle}\overline{\mathtt{S}}\mathtt{\rangle} \prec \mathtt{C\langle}\overline{\mathtt{T}}\mathtt{\rangle}}$ ($\mathtt{S} = \mathtt{C\langle}\overline{\mathtt{S}}\mathtt{\rangle}$, $\mathtt{T} = \mathtt{C\langle}\overline{\mathtt{T}}\mathtt{\rangle}$). By the definition of fields, we have

$\mathtt{class\ C\langle}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{\rangle} \lhd \mathtt{N}\ \mathtt{\{}\overline{\mathtt{V}}\ \overline{\mathtt{f}}\mathtt{;\ ...\}}$

$\mathit{fields}(\mathtt{C\langle}\overline{\mathtt{S}}\mathtt{\rangle}) = \overline{\mathtt{U}}\ \overline{\mathtt{g}}$, $[\overline{\mathtt{S}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}\ \overline{\mathtt{f}}$

$\mathit{fields}(\mathtt{C\langle}\overline{\mathtt{T}}\mathtt{\rangle}) = \overline{\mathtt{U}}\ \overline{\mathtt{g}}$, $[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}\ \overline{\mathtt{f}}$

Then, $\Delta \vdash [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\overline{\mathtt{V}} \prec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}$ by Lemma 23 finishes the case.

Case $\dfrac{\Delta \vdash \mathtt{S} <: \mathtt{U} \qquad \Delta \vdash \mathtt{U} \prec \mathtt{N}}{\Delta \vdash \mathtt{S} \prec \mathtt{dyn\langle N\rangle}}$ ($\mathtt{T} = \mathtt{N}$). Since we have $\mathit{bound}_\Delta(\mathtt{dyn\langle N\rangle}) = \mathit{bound}_\Delta(\mathtt{N})$, the conclusion is immediate from and the induction hypothesis.

□

Lemma 40. If $\Delta \vdash \mathtt{S} \nprec \mathtt{T}$ and $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T})) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$, then $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{S})) = \overline{\mathtt{S}}\ \overline{\mathtt{g}}$ and $\Delta \vdash \mathtt{S}_i \prec \mathtt{T}_i$ and $\mathtt{g}_i = \mathtt{f}_i$ for all $i \leq \#(\overline{\mathtt{f}})$.

Proof. We have

$\Delta \vdash \mathtt{S} <: \mathtt{U} \quad \Delta \vdash \mathtt{U} \prec \mathtt{T}$

for some $\mathtt{U}$. Then the conclusion is immediate from Lemma 38 and Lemma 39. □

Lemma 41. If $\Delta \vdash \mathtt{C\langle}\overline{\mathtt{T}}\mathtt{\rangle}$ ok for some $\mathtt{class\ C\langle}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{\rangle} \lhd \mathtt{N}\ \mathtt{\{...\}}$ under CT $\mathtt{OK\ IN}$ FGJ$^{(\!(}$, then $\Delta \vdash \overline{\mathtt{T}} \nprec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$.

Proof. Since $\dfrac{\begin{array}{c}\mathtt{class\ C\langle}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{\rangle} \lhd \mathtt{N}\ \mathtt{\{...\}} \qquad \Delta \vdash \overline{\mathtt{T}}\ \mathtt{ok} \\ \forall \kappa_i \in \overline{\kappa}.(\ \kappa_i = \Diamond \text{ implies } \mathit{dynfree}_\Delta(\mathtt{T}_i)\ )\end{array}}{\forall \mathtt{T}_i \in \overline{\mathtt{T}}.(\ \Delta \vdash \mathtt{T}_i \nprec [\mathtt{T}_1/\mathtt{X}_1, \ldots, \mathtt{T}_{i-1}/\mathtt{X}_{i-1}]\mathtt{N}_i\ )}{\Delta \vdash \mathtt{C\langle}\overline{\mathtt{T}}\mathtt{\rangle}\ \mathtt{ok}}$, we have

$\Delta \vdash \mathtt{T}_i \nprec [\mathtt{T}_1/\mathtt{X}_1, \ldots, \mathtt{T}_{i-1}/\mathtt{X}_{i-1}]\mathtt{N}_i$

Since $\mathtt{X}_1 <: \mathtt{N}_1, \ldots, \mathtt{X}_{i-1} <: \mathtt{N}_{i-1} \vdash \mathtt{N}_i$ ok by the rule TR-CLASS, $\mathtt{X}_j$ ($j \geq i$) does not appear in $\mathtt{N}_i$ and we have $\Delta \vdash \overline{\mathtt{T}} \nprec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$. □

Lemma 42. If $\Delta \vdash \mathtt{P}$ ok and $mtype(\mathtt{m}, \mathtt{P}) = \overline{\mathtt{U}} \to \mathtt{U}$ under CT OK IN FGJ$^{(\!(}$, then for any $\mathtt{N}$ such that $\Delta \vdash \mathtt{N} <: \mathtt{P}$ and $\Delta \vdash \mathtt{N}$ ok, we have $mtype(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}} \to \mathtt{U}'$ for some $\mathtt{U}'$ such that $\Delta \vdash \mathtt{U}' \precnsim \mathtt{U}$.

Proof. By induction on the derivation of $\Delta \vdash \mathtt{N} <: \mathtt{P}$ with case analysis on the last rule used.

Case $\Delta \vdash \mathtt{N} <: \mathtt{N}$ ($\mathtt{P} = \mathtt{N}$). Immediate from $mtype(\mathtt{m}, \mathtt{N}) = mtype(\mathtt{m}, \mathtt{P})$.

Case $\dfrac{\Delta \vdash \mathtt{N} <: \mathtt{Q} \qquad \Delta \vdash \mathtt{Q} <: \mathtt{P}}{\Delta \vdash \mathtt{N} <: \mathtt{P}}$. Immediate from the induction hypothesis.

Case $\dfrac{\mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \mathtt{\lhd}\overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{Q}\ \mathtt{\{...\}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{T}}\mathtt{>} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{Q}}$ ($\mathtt{N} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$, $\mathtt{P} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{Q}$). We have

$\mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \mathtt{\lhd}\overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{Q}\ \mathtt{\{...}\ \overline{\mathtt{M}}\mathtt{\}}$

If $\overline{\mathtt{M}}$ do not include a declaration of $\mathtt{m}$, then $mtype(\mathtt{m}, \mathtt{N}) = mtype(\mathtt{m}, \mathtt{P})$ by the definition of $mtype$ finishes the case. Otherwise, we have

$mtype(\mathtt{m}, [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{Q}) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}](\overline{\mathtt{U}}' \to \mathtt{U}_0)$

$[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}_0 = \mathtt{U}$

By TR-CLASS, TR-METHOD and Lemma 41, it must be the case that

$\mathtt{U}_0'\ \mathtt{m(}\overline{\mathtt{U}}'\ \overline{\mathtt{x}}\mathtt{)\{\ return\ ...;\ \}} \in \overline{\mathtt{M}}$

$\overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}} \vdash \mathtt{U}_0' <: \mathtt{U}_0 \quad \Delta \vdash \overline{\mathtt{T}} \precnsim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$

By Lemma 28 and Lemma 27, we have

$\Delta \vdash [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}_0' \precnsim \mathtt{U}$

Since $mtype(\mathtt{m}, \mathtt{N}) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}](\overline{\mathtt{U}}' \to \mathtt{U}_0')$, by the definition of $mtype$, letting $\mathtt{U}' = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}_0'$ finishes the case.

□

Lemma 43. If $\Delta \vdash \mathtt{P}$ ok and $mtype(\mathtt{m}, \mathtt{P}) = \overline{\mathtt{U}} \to \mathtt{U}$ under CT OK IN FGJ$^{(\!(}$, then for any $\mathtt{N}$ such that $\Delta \vdash \mathtt{N} \prec \mathtt{P}$ and $\Delta \vdash \mathtt{N}$ ok, we have $mtype(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}}' \to \mathtt{U}'$ for some $\overline{\mathtt{U}}'$ and $\mathtt{U}'$ such that $\Delta \vdash \overline{\mathtt{U}}' \prec \overline{\mathtt{U}}$ and $\Delta \vdash \mathtt{U}' \prec \mathtt{U}$.

Proof. By induction on the derivation of $\Delta \vdash \mathtt{N} \prec \mathtt{P}$ with case analysis on the last rule used.

Case $\Delta \vdash \mathtt{N} \prec \mathtt{N}$ ($\mathtt{P} = \mathtt{N}$). Immediate from $mtype(\mathtt{m}, \mathtt{N}) = mtype(\mathtt{m}, \mathtt{P})$.

Case $\dfrac{\Delta \vdash \mathtt{N} \prec \mathtt{Q} \qquad \Delta \vdash \mathtt{Q} \prec \mathtt{P}}{\Delta \vdash \mathtt{N} \prec \mathtt{P}}$. Immediate from the induction hypothesis.

Case $\dfrac{\Delta \vdash \overline{\mathtt{S}} \prec \overline{\mathtt{T}}}{\Delta \vdash \mathtt{C<}\overline{\mathtt{S}}\mathtt{>} \prec \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}}$ ($\mathtt{N} = \mathtt{C<}\overline{\mathtt{S}}\mathtt{>}$). Suppose $mtype(\mathtt{m}, \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{V}} \to \mathtt{V}$. By the definition of $mtype$, we have

$mtype(\mathtt{m}, \mathtt{C<}\overline{\mathtt{S}}\mathtt{>}) = [\overline{\mathtt{S}}/\overline{\mathtt{X}}](\overline{\mathtt{V}} \to \mathtt{V}) = \overline{\mathtt{U}}' \to \mathtt{U}'$

$mtype(\mathtt{m}, \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}](\overline{\mathtt{V}} \to \mathtt{V}) = \overline{\mathtt{U}} \to \mathtt{U}$

Then the conclusion is immediate from Lemma 23.

□

Lemma 44. If $\Delta \vdash \mathtt{P}$ ok and $mtype(\mathtt{m}, \mathtt{P}) = \overline{\mathtt{U}} \to \mathtt{U}$ under CT OK IN FGJ$^{(\!(}$, then for any $\mathtt{N}$ such that $\Delta \vdash \mathtt{N} \precnsim \mathtt{P}$ and $\Delta \vdash \mathtt{N}$ ok, we have $mtype(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}}' \to \mathtt{U}'$ for some $\overline{\mathtt{U}}'$ and $\mathtt{U}'$ such that $\Delta \vdash \overline{\mathtt{U}}' \prec \overline{\mathtt{U}}$ and $\Delta \vdash \mathtt{U}' \precnsim \mathtt{U}$.

Proof. Immediate from Lemma 42, Lemma 43 and transitivity of $\prec$. □

Lemma 45. Suppose $\Delta \vdash \mathtt{N}$ ok and $\Delta \vdash \mathtt{N} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<}\overline{\mathtt{X}}\mathtt{>}$ and $mtype(\mathtt{m}, \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U}$ and $mtype(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}}' \to \mathtt{U}'$ under CT OK IN FGJ$^{(\!(}$. If $[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}_i = \mathtt{U}_i$, then $\mathtt{U}_i' = \mathtt{U}_i$.

Proof. Suppose $mtype(\mathtt{m}, \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) = \overline{\mathtt{V}} \to \mathtt{V}$. By Lemma 42, we have

$\overline{\mathtt{V}} = \overline{\mathtt{U}}'$

Then, by the assumption, we have $\mathtt{U}_i = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}_i = \mathtt{V}_i = \mathtt{U}_i'$. □

Lemma 46. Suppose $\Delta \vdash \mathtt{N}$ ok and $\Delta \vdash \mathtt{N} \precnsim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<}\overline{\mathtt{X}}\mathtt{>}$ and $mtype(\mathtt{m}, \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U}$ and $mtype(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}}' \to \mathtt{U}'$ under CT OK IN FGJ$^{(\!(}$. If $[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}_i = \mathtt{U}_i$, then $\mathtt{U}_i' = \mathtt{U}_i$.

Proof. We have

$\Delta \vdash \mathtt{N} <: \mathtt{C<}\overline{\mathtt{S}}\mathtt{>} \quad \Delta \vdash \mathtt{C<}\overline{\mathtt{S}}\mathtt{>} \prec \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$

for some $\overline{\mathtt{S}}$. Since $\overline{\mathtt{T}}$ do not contain $\mathtt{X}_j$ by the rule TR-CLASS, we have

$[\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{U}_i = \mathtt{U}_i$

Then, the conclusion is immediate from Lemma 45. □

Lemma 47. If $mtype(\mathtt{m},\ \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U}$ and $mtype(\mathtt{m},\ \mathtt{N}) = \overline{\mathtt{U}}' \to \mathtt{U}'$ and $tyargs(\mathtt{N},\ \mathtt{C}) = \overline{\mathtt{S}}$, then $[\overline{\mathtt{S}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} = \overline{\mathtt{U}}'$.

Proof. By induction on the derivation of $tyargs(\mathtt{N},\ \mathtt{C}) = \overline{\mathtt{S}}$ with a case analysis on the last rule used.

Case $tyargs(\mathtt{C<}\overline{\mathtt{S}}\mathtt{>},\ \mathtt{C}) = \overline{\mathtt{S}}$ $(\mathtt{N} = \mathtt{C<}\overline{\mathtt{S}}\mathtt{>})$. Immediate from the definition of $mtype$.

Case $\dfrac{\bullet \vdash \mathtt{N} <: \mathtt{P} \qquad tyargs(\mathtt{P},\ \mathtt{C}) = \overline{\mathtt{S}}}{tyargs(\mathtt{N},\ \mathtt{C}) = \overline{\mathtt{S}}}$. By the induction hypothesis, we have

$mtype(\mathtt{m},\ \mathtt{P}) = \overline{\mathtt{U}}'' \to \mathtt{U}'' \quad [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} = \overline{\mathtt{U}}''$

Then, by Lemma 42, we have $\overline{\mathtt{U}}' = \overline{\mathtt{U}}''$.

□

Lemma 48. Under CT OK IN FGJ$^{(\mathbb{D})}$, if $\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{T}$ and $\Delta_1 \vdash \overline{\mathtt{U}} \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$ where $\Delta_1 \vdash \overline{\mathtt{U}}$ ok and none of $\overline{\mathtt{X}}$ appears in Δ_1 and $\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]$ ok, then $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e} : \mathtt{S}$ for some $\mathtt{S}$ such that $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S} \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$.

Proof. By induction on the derivation of $\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{T}$ with a case analysis on the last rule used.

Case TR-VAR.

$\mathtt{e} = \mathtt{x} \quad \mathtt{T} = \Gamma(\mathtt{x})$

By Lemma 31, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T} \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$

Then, $\mathtt{S} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T} = ([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma)(\mathtt{x})$ finishes the case.

Case TR-FIELD1.

$\mathtt{e} = \mathtt{e}_0.\mathtt{f}_i \quad \Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$

$fields(bound_{\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2}(\mathtt{T}_0)) = \overline{\mathtt{T}\ \mathtt{f}} \quad \mathtt{T} = \mathtt{T}_i$

By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \mathtt{S}_0 \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}_0 \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0$

for some $\mathtt{S}_0$. By Lemma 34, Lemma 37 and Lemma 26, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash bound_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0) \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}](bound_{\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2}(\mathtt{T}_0))$

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash bound_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0) \precsim bound_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0)$

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash bound_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0) \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}](bound_{\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2}(\mathtt{T}_0))$

By Lemma 40, $fields(bound_{\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0)) = \overline{\mathtt{S}\ \mathtt{g}}$ and we have $\mathtt{f}_j = \mathtt{g}_j$ and $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}_j \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_j$ for $j \leq \#(\overline{\mathtt{f}})$.

By the rule TR-FIELD1, we have $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0.\mathtt{f}_i : \mathtt{S}_i$. Letting $\mathtt{S} = \mathtt{S}_i$ finishes the case.

Case TR-FIELD2.

$\mathtt{e} = \mathtt{get(e_0, f)} \quad \Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \mathtt{T} = \mathtt{dyn<Object>}$

By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \mathtt{T}'_0 \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{T}'_0 \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0$

for some $\mathtt{T}'_0$. Then, by TR-FIELD2, $\mathtt{S} = \mathtt{dyn<Object>}$ finishes the case.

Case TR-INVK1.

$\mathtt{e} = \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \quad \Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$

$\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad bound_{\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2}(\mathtt{T}_0) = \mathtt{Q} \quad \Delta \vdash \mathtt{Q} \precsim [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{C<}\overline{\mathtt{Y}}\mathtt{>}$

$dynfree_{\Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2}(\mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) \quad mtype(\mathtt{m},\ \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) = \overline{\mathtt{V}}' \to \mathtt{T} \quad \Delta_1, \overline{\mathtt{X}^{\overline{\iota}} <: \mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{V}} \precsim \overline{\mathtt{V}}'$

By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \mathtt{S}_0 \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}_0 \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0$

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{e}} : \overline{\mathtt{S}} \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{S}} \precsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}$

for some $\mathtt{S}_0$. By Lemma 34, Lemma 37 and Lemma 26, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0) \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}](\mathit{bound}_{\Delta_1,\overline{\mathtt{X}}^{\overline{\iota}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T}_0))$
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0) \precnsim \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0)$
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0) \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}](\mathit{bound}_{\Delta_1,\overline{\mathtt{X}}^{\overline{\iota}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T}_0))$

Let $\mathtt{N} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$. By Lemma 31 and Lemma 26, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}](\mathit{bound}_{\Delta_1,\overline{\mathtt{X}}^{\overline{\iota}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T}_0)) \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}$
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0) \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}$

Since $\mathit{dynfree}_{\Delta_1,\overline{\mathtt{X}}^{\overline{\iota}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{N})$, we have

$\mathit{dynfree}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N})$

Let $\mathtt{P} = \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0)$. By Lemma 5, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{P} <: [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}$

Then, by Lemma 42, we have

$\mathit{mtype}(\mathtt{m}, \mathtt{P}) = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}' \to \mathtt{T}' \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{T}' \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$

By Lemma 31 and Lemma 26, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}} \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}' \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{S}} \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}'$

Then, by the rule TR-INVK1,

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0\mathtt{.m(}\overline{\mathtt{e}}\mathtt{)} : \mathtt{T}'$

finishes the case.

Case TR-INVK2.

$\mathtt{e} = \mathtt{e}_0\mathtt{.m[}\overline{\mathtt{V}}'\mathtt{|C<}\overline{\mathtt{Y}}\mathtt{>]\,(}\overline{\mathtt{e}}\mathtt{)} \quad \Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathit{bound}_{\Delta_1,\overline{\mathtt{X}}^{\overline{\iota}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T}_0) = \mathtt{N}$
$\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{N} \precnsim [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{C<}\overline{\mathtt{Y}}\mathtt{>} \quad \mathit{mtype}(\mathtt{m}, \mathtt{C<}\overline{\mathtt{Y}}\mathtt{>}) = \overline{\mathtt{V}}' \to \mathtt{U}$
$\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{V}} \precnsim [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{V}}' \quad \mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{U}$

By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \mathtt{S}_0 \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{S}_0 \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0$
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{e}} : \overline{\mathtt{S}} \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{S}} \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}$

for some $\mathtt{S}_0$. By Lemma 34, Lemma 37 and Lemma 26, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0) \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}](\mathit{bound}_{\Delta_1,\overline{\mathtt{X}}^{\overline{\iota}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T}_0))$
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0) \precnsim \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}([\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0)$
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0) \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}](\mathit{bound}_{\Delta_1,\overline{\mathtt{X}}^{\overline{\iota}}<:\overline{\mathtt{N}},\Delta_2}(\mathtt{T}_0))$

By Lemma 31 and Lemma 26, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N} \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{C<}\overline{\mathtt{Y}}\mathtt{>} \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{P} \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{C<}\overline{\mathtt{Y}}\mathtt{>}$
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}} \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{V}}' \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{S}} \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{V}}'$

where $\mathtt{P} = \mathit{bound}_{\Delta_1,[\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2}(\mathtt{S}_0)$. Then, by the rule TR-INVK2,

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0\mathtt{.m[}\overline{\mathtt{V}}'\mathtt{|C<}\overline{\mathtt{Y}}\mathtt{>]\,(}\overline{\mathtt{e}}\mathtt{)} : \mathtt{S}$

where $\mathtt{S} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}][\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{U} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$ finishes the case.

Case TR-INVK3.

$\mathtt{e} = \mathtt{invoke(}\mathtt{e}_0\mathtt{,\,m,}\,\overline{\mathtt{e}}\mathtt{)} \quad \Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathtt{T} = \mathtt{dyn<Object>}$

By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{e}_0 : \mathtt{T}'_0 \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{T}'_0 \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}_0$
$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{e}} : \overline{\mathtt{V}}' \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{V}}' \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}$

for some $\mathtt{T}'_0$ and $\overline{\mathtt{V}}'$. Then, by TR-INVK3, $\mathtt{S} = \mathtt{dyn<Object>}$ finishes the case.

Case TR-NEW.

$\mathtt{e} = \mathtt{new\ N(}\overline{\mathtt{e}}\mathtt{)} \quad \Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \mathtt{N}\ \mathrm{ok}$
$\mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \quad \Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$
$\Delta_1, \overline{\mathtt{X}}^{\overline{\iota}} <: \overline{\mathtt{N}}, \Delta_2 \vdash \overline{\mathtt{V}} \precnsim \overline{\mathtt{T}} \quad \mathtt{T} = \mathtt{N}$

By Lemma 33, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}\ \mathrm{ok}$

By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{e}} : \overline{\mathtt{V}}' \quad \Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{V}}' \precnsim [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}}$

By Lemma 31 and Lemma 26, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{V}} \precdot [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}$ $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \overline{\mathtt{V}}' \precdot [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\overline{\mathtt{T}}$

Then, by TR-NEW, $\mathtt{S} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{N}$ finishes the case.

Case TR-CAST.

$\mathtt{e} = (\!|\mathtt{T}|\!)\,\mathtt{e}_0$ $\Delta_1, \overline{\mathtt{X}^{\mathtt{T}}} <: \overline{\mathtt{N}}, \Delta_2; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}$

By the induction hypothesis, we have

$\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2; [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}'$ $\Delta_1, [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\Delta_2 \vdash \mathtt{V}' \precdot [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{V}$

for some $\mathtt{V}'$. Then, by TR-CAST, $\mathtt{S} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$ finishes the case.

Case TR-ERROR.

$\mathtt{e} = \mathtt{Error}[\mathcal{E}]$

Immediate from the rule TR-ERROR by letting $\mathtt{S} = [\overline{\mathtt{U}}/\overline{\mathtt{X}}]\mathtt{T}$.

□

Lemma 49. Under CT OK IN FGJ$^{(\!|\,|\!)}$, if $\Delta; \Gamma, \overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{T}$ and $\Delta; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{d}} : \overline{\mathtt{S}}$ where $\Delta \vdash \overline{\mathtt{S}} \precdot \overline{\mathtt{T}}$, then $\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e} : \mathtt{S}$ for some $\mathtt{S}$ such that $\Delta \vdash \mathtt{S} \precdot \mathtt{T}$.

Proof. By induction on the derivation of $\Delta; \Gamma, \overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{T}$ with a case analysis on the last rule used.

Case TR-VAR.

$\mathtt{e} = \mathtt{x}$

If $\mathtt{x} \in \mathit{dom}(\Gamma)$, the conclusion is immediate, since $[\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e} = \mathtt{x}$ and $\mathtt{S} = \mathtt{T}$. On the other hand, if $\mathtt{x} = \mathtt{x}_i$ and $\mathtt{T} = \mathtt{T}_i$, then letting $\mathtt{S} = \mathtt{S}_i$ finishes the case.

Case TR-FIELD1.

$\mathtt{e} = \mathtt{e}_0.\mathtt{f}_i$ $\Delta; \Gamma, \overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$

$\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{S}}\ \overline{\mathtt{f}}$ $\mathtt{T} = \mathtt{S}_i$

By the induction hypothesis, we have

$\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{T}_0'$

for some $\mathtt{T}_0'$ such that $\Delta \vdash \mathtt{T}_0' \precdot \mathtt{T}_0$. By Lemma 40, $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0')) = \overline{\mathtt{U}}\ \overline{\mathtt{g}}$ such that $\Delta \vdash \mathtt{U}_j \precdot \mathtt{S}_j$ and $\mathtt{g}_j = \mathtt{f}_j$ for all $j \leq \#(\overline{\mathtt{S}})$. Then, by the rule TR-FIELD1, we have

$\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0.\mathtt{f}_i : \mathtt{U}_i$

and letting $\mathtt{S} = \mathtt{U}_i$ finishes the case.

Case TR-FIELD2.

$\mathtt{e} = \mathtt{get(e_0, f_i)}$ $\Delta; \Gamma, \overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$

$\mathtt{T} = \mathtt{dyn\texttt{<}Object\texttt{>}}$

By the induction hypothesis, we have

$\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{T}_0'$

for some $\mathtt{T}_0'$ such that $\Delta \vdash \mathtt{T}_0' \precdot \mathtt{T}_0$. Then, by the rule TR-FIELD2, we have

$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{get}([\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0, \mathtt{f}_i) : \mathtt{dyn\texttt{<}Object\texttt{>}}$

and letting $\mathtt{S} = \mathtt{dyn\texttt{<}Object\texttt{>}}$ finishes the case.

Case TR-INVK1.

$\mathtt{e} = \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}})$ $\Delta; \Gamma, \overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$

$\Delta; \Gamma, \overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$ $\mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{Q}$ $\Delta \vdash \mathtt{Q} \precdot \mathtt{N}$

$\mathit{dynfree}_\Delta(\mathtt{N})$ $\mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{T}} \to \mathtt{T}$ $\Delta \vdash \overline{\mathtt{V}} \precdot \overline{\mathtt{T}}$

By the induction hypothesis, we have

$\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{T}_0'$ $\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}} : \overline{\mathtt{V}}'$

for some $\mathtt{T}_0'$ and $\overline{\mathtt{V}}'$ such that $\Delta \vdash \mathtt{T}_0' \precdot \mathtt{T}_0$ and $\Delta \vdash \overline{\mathtt{V}}' \precdot \overline{\mathtt{V}}$. By Lemma 37 and Lemma 26, we have

$\mathit{bound}_\Delta(\mathtt{T}_0') = \mathtt{P}$ $\Delta \vdash \mathtt{P} \precdot \mathtt{Q}$ $\Delta \vdash \mathtt{P} \precdot \mathtt{N}$ $\Delta \vdash \overline{\mathtt{V}}' \precdot \overline{\mathtt{T}}$

Then, by the rule TR-INVK1, we have

$\Delta; \Gamma \vdash_{\mathrm{R}} ([\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0).\mathtt{m}([\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}) : \mathtt{T}$

finishes the case.

Case TR-INVK2.

$\mathtt{e} = \mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C\texttt{<}}\overline{\mathtt{X}}\mathtt{\texttt{>}}](\overline{\mathtt{e}})$ $\Delta; \Gamma, \overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$

$\Delta; \Gamma, \overline{\mathtt{x}}:\overline{\mathtt{T}} \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$ $\mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N}$ $\Delta \vdash \mathtt{N} \precdot [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{C\texttt{<}}\overline{\mathtt{X}}\mathtt{\texttt{>}}$

$\mathit{mtype}(\mathtt{m}, \mathtt{C\texttt{<}}\overline{\mathtt{X}}\mathtt{\texttt{>}}) = \overline{\mathtt{U}} \to \mathtt{U}$ $\Delta \vdash \overline{\mathtt{V}} \precdot [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}$ $\mathtt{T} = [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{U}$

By the induction hypothesis, we have

$\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{T}_0'$ $\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}} : \overline{\mathtt{V}}'$
for some $\mathtt{T}_0'$ and $\overline{\mathtt{V}}'$ such that $\Delta \vdash \mathtt{T}_0' \precsim \mathtt{T}_0$ and $\Delta \vdash \overline{\mathtt{V}}' \precsim \overline{\mathtt{V}}$. By Lemma 37 and Lemma 26, we have
$bound_\Delta(\mathtt{T}_0') = \mathtt{P}$ $\Delta \vdash \mathtt{P} \precsim [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{C<}\overline{\mathtt{X}}\mathtt{>}$ $\Delta \vdash \overline{\mathtt{V}}' \precsim [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}$
Then, by the rule TR-INVK2, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} ([\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0)\mathtt{.m[}\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>]}([\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}) : [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{U}$
and letting $\mathtt{S} = \mathtt{T} = [\overline{\mathtt{S}}/\overline{\mathtt{X}}]\mathtt{U}$ finishes the case.

Case TR-INVK3.
$\mathtt{e} = \mathtt{invoke(e_0, m, }\overline{\mathtt{e}}\mathtt{)}$ $\Delta; \Gamma, \overline{\mathtt{x}} : \overline{\mathtt{T}} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta; \Gamma, \overline{\mathtt{x}} : \overline{\mathtt{T}} \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$ $\mathtt{T} = \mathtt{dyn<Object>}$
By the induction hypothesis, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{T}_0'$ $\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}} : \overline{\mathtt{V}}'$
for some $\mathtt{T}_0'$ and $\overline{\mathtt{V}}'$ such that $\Delta \vdash \mathtt{T}_0' \precsim \mathtt{T}_0$ and $\Delta \vdash \overline{\mathtt{V}}' \precsim \overline{\mathtt{V}}$. Then, by the rule TR-INVK3, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{invoke(}[\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0\mathtt{, m, }[\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}\mathtt{) : dyn<Object>}$
and letting $\mathtt{S} = \mathtt{dyn<Object>}$ finishes the case.

Case TR-NEW.
$\mathtt{e} = \mathtt{new\ N(}\overline{\mathtt{e}}\mathtt{)}$ $\Delta \vdash \mathtt{N}$ ok
$\mathit{fields}(\mathtt{N}) = \overline{\mathtt{S}}\ \overline{\mathtt{f}}$ $\Delta; \Gamma, \overline{\mathtt{x}} : \overline{\mathtt{T}} \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{U}}$
$\Delta \vdash \overline{\mathtt{U}} \precsim \overline{\mathtt{S}}$ $\mathtt{T} = \mathtt{N}$
By the induction hypothesis, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}} : \overline{\mathtt{U}}'$
for some $\overline{\mathtt{U}}'$ such that $\Delta \vdash \overline{\mathtt{U}}' \precsim \overline{\mathtt{U}}$. By Lemma 26, we have
$\Delta \vdash \overline{\mathtt{U}}' \precsim \overline{\mathtt{S}}$
Then, by the rule TR-NEW, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{new\ N(}[\overline{\mathtt{d}}/\overline{\mathtt{x}}]\overline{\mathtt{e}}\mathtt{) : N}$
and letting $\mathtt{S} = \mathtt{N}$ finishes the case.

Case TR-CAST.
$\mathtt{e} = (\!|\mathtt{T}|\!)\,\mathtt{e}_0$ $\Delta; \Gamma, \overline{\mathtt{x}} : \overline{\mathtt{T}} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
By the induction hypothesis, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0 : \mathtt{T}_0'$
for some $\mathtt{T}_0'$ such that $\Delta \vdash \mathtt{T}_0' \precsim \mathtt{T}_0$. Then, by the rule TR-CAST, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} (\!|\mathtt{T}|\!)\,([\overline{\mathtt{d}}/\overline{\mathtt{x}}]\mathtt{e}_0) : \mathtt{T}$
and letting $\mathtt{S} = \mathtt{T}$ finishes the case.

Case TR-ERROR.
$\mathtt{e} = \mathtt{Error}[\mathcal{E}]$
Immediate from the rule TR-ERROR by letting $\mathtt{S} = \mathtt{T}$.

□

Lemma 50. If $\mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}} \to \mathtt{U}$ and $\mathit{mbody}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0$ where $\Delta \vdash \mathtt{N}$ ok under CT OK IN FGJ$^{(\!|\,|\!)}$, then there exist some P and V such that $\Delta \vdash \mathtt{N} \precsim \mathtt{P}$ and $\Delta \vdash \mathtt{P}$ ok and $\Delta \vdash \mathtt{V} \precsim \mathtt{U}$ and $\Delta; \overline{\mathtt{x}} : \overline{\mathtt{U}}, \mathtt{this} : \mathtt{P} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}$.

Proof. By induction on the derivation of $\mathit{mbody}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0$ using Lemma 31 and Lemma 48.

Case $\dfrac{\mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{Q}\ \{\ldots;\quad \overline{\mathtt{M}}\} \qquad \mathtt{T}_0\ \mathtt{m(}\overline{\mathtt{S}}\ \overline{\mathtt{x}}\mathtt{)\{\ return\ e;\ \}} \in \overline{\mathtt{M}}}{\mathit{mbody}(\mathtt{m}, \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) = \overline{\mathtt{x}}.[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{e}}$.
$\mathtt{e}_0 = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{e}$ $\mathtt{N} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$.
Let $\Gamma = \overline{\mathtt{x}} : \overline{\mathtt{S}}, \mathtt{this} : \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}$ and $\Delta' = \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}}$. By TR-CLASS and TR-METHOD, we have $\Delta'; \Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{S}_0$ and $\Delta' \vdash \mathtt{S}_0 \precsim \mathtt{T}_0$ for some $\mathtt{S}_0$. By $\Delta \vdash \mathtt{N}$ ok, we have
$\Delta \vdash \overline{\mathtt{T}}$ ok $\Delta' \vdash [\overline{\mathtt{T}}/\overline{\mathtt{X}}]$ ok
By Lemma 41, we have
$\Delta \vdash \overline{\mathtt{T}} \precsim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{N}}$
By Lemma 27, Lemma 31 and Lemma 48,
$\Delta \vdash [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{S}_0 \precsim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{T}_0$ $\Delta; \overline{\mathtt{x}} : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{S}}, \mathtt{this} : \mathtt{C<}\overline{\mathtt{T}}\mathtt{>} \vdash_{\mathrm{R}} [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{e} : \mathtt{S}_0'$

where $\Delta \vdash \mathtt{S}'_0 \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{S}_0$. By, $\dfrac{\begin{array}{c}\mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}}\mathtt{\triangleleft}\overline{\mathtt{N}}\mathtt{>\ \triangleleft\ Q\ \{\ldots;\quad \overline{M}\}}\\ \mathtt{S\ m(}\overline{\mathtt{S}}\ \overline{\mathtt{x}}\mathtt{)\{\ return\ e;\ \}} \in \overline{\mathtt{M}}\end{array}}{mtype(\mathtt{m},\ \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}](\overline{\mathtt{S}} \to \mathtt{S})}$, we have

$\overline{\mathtt{U}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{S}} \quad \mathtt{U} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{T}_0$

By Lemma 26, $\mathtt{V} = \mathtt{S}'_0$ and $\mathtt{P} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$ finishes the case.

Case $\dfrac{\begin{array}{c}\mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}}\mathtt{\triangleleft}\overline{\mathtt{N}}\mathtt{>\ \triangleleft\ Q\ \{\ldots;\quad \overline{M}\}}\\ \mathtt{m} \notin \overline{\mathtt{M}} \qquad mbody(\mathtt{m},\ [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{Q}) = \overline{\mathtt{x}}.\mathtt{e}\end{array}}{mbody(\mathtt{m},\ \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) = \overline{\mathtt{x}}.\mathtt{e}}$.

$\mathtt{e}_0 = \mathtt{e} \quad \mathtt{N} = \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}$

Immediate from the induction hypothesis and the fact that $\Delta \vdash \mathtt{C<}\overline{\mathtt{T}}\mathtt{>} <: [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{Q}$.

□

Theorem 51. Under CT OK IN FGJ$^{(\mathbb{D})}$, if $\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e} : \mathtt{T}$ and $\mathtt{e} \longrightarrow \mathtt{e}'$, then $\Delta \vdash \mathtt{T}' \preccurlyeq \mathtt{T}$ and $\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}' : \mathtt{T}'$ for some $\mathtt{T}'$.

Proof. By induction on $\mathtt{e} \longrightarrow \mathtt{e}'$ with a case analysis on the reduction rule used.

Case R-FIELD1.

$\mathtt{e} = \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{).f}_i \quad \mathtt{e}' = \mathtt{v}_i \quad fields(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$

By TR-FIELD1 and TR-NEW, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{)} : \mathtt{N} \quad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{v}} : \overline{\mathtt{U}} \quad \Delta \vdash \overline{\mathtt{U}} \preccurlyeq \overline{\mathtt{T}}$

In particular, $\mathtt{T}' = \mathtt{U}_i$, which satisfies

$\Delta \vdash \mathtt{U}_i \preccurlyeq \mathtt{T}_i \quad \Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{v}_i : \mathtt{U}_i$

finishes the case.

Case R-FIELD2.

$\mathtt{e} = \mathtt{get(new\ N(}\overline{\mathtt{v}}\mathtt{)\ ,\ f}_i\mathtt{)} \quad \mathtt{e}' = \mathtt{v}_i \quad fields(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$

By TR-FIELD2 and TR-NEW, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{)} : \mathtt{N} \quad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{v}} : \overline{\mathtt{U}} \quad \mathtt{T} = \mathtt{dyn<Object>}$

In particular, $\mathtt{T}' = \mathtt{U}_i$, which satisfies

$\Delta \vdash \mathtt{U}_i \preccurlyeq \mathtt{dyn<Object>} \quad \Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{v}_i : \mathtt{U}_i$

finishes the case.

Case R-INVK1.

$\mathtt{e} = \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{).m(}\overline{\mathtt{w}}\mathtt{)} \quad mbody(\mathtt{m},\ \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0 \quad \mathtt{e}' = [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{)/this}]\mathtt{e}_0$

By TR-INVK1 and TR-NEW, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{)} : \mathtt{N} \quad dynfree_{\Delta}(\mathtt{P}) \quad \Delta \vdash \mathtt{N} \preccurlyeq \mathtt{P} \quad mtype(\mathtt{m},\ \mathtt{P}) = \overline{\mathtt{U}} \to \mathtt{T}$

$\Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{w}} : \overline{\mathtt{V}} \quad \Delta \vdash \overline{\mathtt{V}} \preccurlyeq \overline{\mathtt{U}} \quad \Delta \vdash \mathtt{N}$ ok

By Lemma 5 and Lemma 42, we have

$\Delta \vdash \mathtt{N} <: \mathtt{P} \quad mtype(\mathtt{m},\ \mathtt{P}) = \overline{\mathtt{U}} \to \mathtt{U} \quad \Delta \vdash \mathtt{U} \preccurlyeq \mathtt{T}$

By Lemma 50, we have

$\Delta \vdash \mathtt{N} \preccurlyeq \mathtt{P} \quad \Delta \vdash \mathtt{P}$ ok $\quad \Delta \vdash \mathtt{V} \preccurlyeq \mathtt{U} \quad \Delta;\overline{\mathtt{x}} : \overline{\mathtt{U}}, \mathtt{this} : \mathtt{P} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}$

for some $\mathtt{P}$ and $\mathtt{V}$. Then, by Lemma 49, $\Delta;\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{)/this}]\mathtt{e}_0 : \mathtt{T}'$ for some $\mathtt{T}'$ such that $\Delta \vdash \mathtt{T}' \preccurlyeq \mathtt{V}$ and $\Delta \vdash \mathtt{T}' \preccurlyeq \mathtt{T}$ by Lemma 26.

Case R-INVK2.

$\mathtt{e} = \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{).m[}\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>](}\overline{\mathtt{w}}\mathtt{)} \quad mbody(\mathtt{m},\ \mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0$

$\overline{\mathtt{w}} = \mathtt{new}\ \overline{\mathtt{P}}\mathtt{(...)} \quad \bullet \vdash \overline{\mathtt{P}} \preccurlyeq [tyargs(\mathtt{N},\ \mathtt{C})/\overline{\mathtt{X}}]\overline{\mathtt{U}} \quad \mathtt{e}' = [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{)/this}]\mathtt{e}_0$

By TR-INVK2 and TR-NEW, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{new\ N(}\overline{\mathtt{v}}\mathtt{)} : \mathtt{N} \quad \Delta \vdash \mathtt{N} \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<}\overline{\mathtt{X}}\mathtt{>} \quad mtype(\mathtt{m},\ \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U}$

$\Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{w}} : \overline{\mathtt{P}} \quad \Delta \vdash \overline{\mathtt{P}} \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \quad \Delta \vdash \mathtt{N}$ ok $\quad \mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$

By Lemma 44, we have

$mtype(\mathtt{m},\ \mathtt{N}) = \overline{\mathtt{U}}' \to \mathtt{U}' \quad \Delta \vdash \overline{\mathtt{U}}' \prec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \quad \Delta \vdash \mathtt{U}' \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$

for some $\overline{\mathtt{U}}'$ and $\mathtt{U}'$. By Lemma 50 and Lemma 26, we have

$\Delta \vdash \mathtt{N} \preccurlyeq \mathtt{P} \quad \Delta \vdash \mathtt{P}$ ok $\quad \Delta \vdash \mathtt{V} \preccurlyeq \mathtt{U}' \quad \Delta \vdash \mathtt{V} \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$

$\Delta;\overline{\mathtt{x}} : \overline{\mathtt{U}}', \mathtt{this} : \mathtt{P} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}$

for some $\mathtt{P}$ and $\mathtt{V}$. By Lemma 47, we have

$[\mathit{tyargs}(\mathtt{N},\mathtt{C})/\overline{\mathtt{X}}]\overline{\mathtt{U}} = \overline{\mathtt{U}}'$ $\Delta \vdash \overline{\mathtt{P}} \preccurlyeq \overline{\mathtt{U}}'$

Then, by Lemma 49, $\Delta;\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N}(\overline{\mathtt{v}})/\mathtt{this}]\mathtt{e}_0 : \mathtt{T}'$ for some $\mathtt{T}'$ such that $\Delta \vdash \mathtt{T}' \preccurlyeq \mathtt{V}$ and $\Delta \vdash \mathtt{T}' \preccurlyeq \mathtt{T}$ by Lemma 26.

Case R-INVK3.

$\mathtt{e} = \mathtt{invoke(new\ N}(\overline{\mathtt{v}})\mathtt{, m,}\ \overline{\mathtt{w}}\mathtt{)}$ $\mathit{mbody}(\mathtt{m},\mathtt{N}) = \overline{\mathtt{x}}.\mathtt{e}_0$

$\overline{\mathtt{w}} = \mathtt{new}\ \overline{\mathtt{P}}\mathtt{(...)}$ $\bullet \vdash \overline{\mathtt{P}} \preccurlyeq \overline{\mathtt{U}}$

$\mathit{mtype}(\mathtt{m},\mathtt{N}) = \overline{\mathtt{U}} \to \mathtt{U}$ $\mathtt{e}' = [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N}(\overline{\mathtt{v}})/\mathtt{this}]\mathtt{e}_0$

By TR-INVK3 and TR-NEW, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{new\ N}(\overline{\mathtt{e}}) : \mathtt{N}$ $\Delta \vdash \mathtt{N}$ ok $\mathtt{T} = \mathtt{dyn<Object>}$

By, Lemma 50,

$\Delta \vdash \mathtt{N} \preccurlyeq \mathtt{P}$ $\Delta \vdash \mathtt{P}$ ok $\Delta \vdash \mathtt{V} \preccurlyeq \mathtt{U}$ $\Delta \vdash \mathtt{V}$ ok

$\Delta;\overline{\mathtt{x}} : \overline{\mathtt{U}}, \mathtt{this} : \mathtt{P} \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}$

for some $\mathtt{P}$ and $\mathtt{V}$. Then, by Lemma 49, we have

$\Delta \vdash \mathtt{T}' \preccurlyeq \mathtt{V}$ $\Delta;\Gamma \vdash_{\mathrm{R}} [\overline{\mathtt{w}}/\overline{\mathtt{x}}, \mathtt{new\ N}(\overline{\mathtt{v}})/\mathtt{this}]\mathtt{e}_0 : \mathtt{T}'$

for some $\mathtt{T}'$ such that $\Delta \vdash \mathtt{T}' \preccurlyeq \mathtt{dyn<Object>}$.

Case R-CAST.

$\mathtt{e} = (\!|\mathtt{T}|\!)\,\mathtt{new\ N}(\overline{\mathtt{v}})$ $\bullet \vdash \mathtt{N} \preccurlyeq \mathtt{T}$ $\mathtt{e}' = \mathtt{new\ N}(\overline{\mathtt{v}})$

By TR-CAST and TR-NEW, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} (\!|\mathtt{T}|\!)\,\mathtt{new\ N}(\overline{\mathtt{v}}) : \mathtt{T}$ $\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{new\ N}(\overline{\mathtt{v}}) : \mathtt{N}$

Then, $\mathtt{T}' = \mathtt{N}$ finishes the case.

Case RC-FIELD1.

$\mathtt{e} = \mathtt{e}_0.\mathtt{f}$ $\mathtt{e}' = \mathtt{e}_0'.\mathtt{f}$ $\mathtt{e}_0 \longrightarrow \mathtt{e}_0'$

By TR-FIELD1, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$ $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$ $\mathtt{T} = \mathtt{T}_i$

By the induction hypothesis, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0' : \mathtt{T}_0'$

for some $\mathtt{T}_0'$ such that $\Delta \vdash \mathtt{T}_0' \preccurlyeq \mathtt{T}_0$. By Lemma 40, $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0')) = \overline{\mathtt{S}}\ \overline{\mathtt{g}}$ and, for $j \leq \#(\overline{\mathtt{f}})$, we have $\mathtt{g}_i = \mathtt{f}_i$ and $\Delta \vdash \mathtt{S}_i \preccurlyeq \mathtt{T}_i$. Therefore, by the rule TR-FIELD1, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0'.\mathtt{f} : \mathtt{S}_i$

Then, letting $\mathtt{T}' = \mathtt{S}_i$ finishes the case.

Case RC-FIELD2.

$\mathtt{e} = \mathtt{get(e}_0\mathtt{, f)}$ $\mathtt{e}' = \mathtt{get(e}_0'\mathtt{, f)}$ $\mathtt{e}_0 \longrightarrow \mathtt{e}_0'$

By TR-FIELD2, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$ $\mathtt{T} = \mathtt{dyn<Object>}$

By the induction hypothesis, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0' : \mathtt{T}_0'$

for some $\mathtt{T}_0'$ such that $\Delta \vdash \mathtt{T}_0' \preccurlyeq \mathtt{T}_0$. Therefore, by the rule TR-FIELD2, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{get(e}_0'\mathtt{, f)} : \mathtt{dyn<Object>}$

Then, letting $\mathtt{T}' = \mathtt{dyn<Object>}$ finishes the case.

Case RC-INVK-RECV1.

$\mathtt{e} = \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}})$ $\mathtt{e}' = \mathtt{e}_0'.\mathtt{m}(\overline{\mathtt{e}})$ $\mathtt{e}_0 \longrightarrow \mathtt{e}_0'$

By TR-INVK1, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$ $\Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}}$ $\mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{Q}$ $\Delta \vdash \mathtt{Q} \preccurlyeq \mathtt{N}$

$\mathit{dynfree}_\Delta(\mathtt{N})$ $\mathit{mtype}(\mathtt{m},\mathtt{N}) = \overline{\mathtt{U}} \to \mathtt{T}$ $\Delta \vdash \overline{\mathtt{V}} \preccurlyeq \overline{\mathtt{U}}$

By the induction hypothesis, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0' : \mathtt{T}_0'$

for some $\mathtt{T}_0'$ such that $\Delta \vdash \mathtt{T}_0' \preccurlyeq \mathtt{T}_0$. By Lemma 37, Lemma 26, we have

$\mathit{bound}_\Delta(\mathtt{T}_0') = \mathtt{P}$ $\Delta \vdash \mathtt{P} \preccurlyeq \mathtt{N}$

Then, by the rule TR-INVK1, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0'.\mathtt{m}(\overline{\mathtt{e}}) : \mathtt{T}$

Case RC-INVK-ARG1.

$\mathtt{e} = \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{v}}\mathtt{,}\ \mathtt{e}_i\mathtt{,}\ \overline{\mathtt{e}})$ $\mathtt{e}' = \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{v}}\mathtt{,}\ \mathtt{e}_i'\mathtt{,}\ \overline{\mathtt{e}})$ $\mathtt{e}_i \longrightarrow \mathtt{e}_i'$

By TR-INVK1, we have

$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{v}}, \mathtt{e}_i, \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{Q} \quad \Delta \vdash \mathtt{Q} \preccurlyeq \mathtt{N}$
$\mathit{dynfree}_\Delta(\mathtt{N}) \quad \mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}} \to \mathtt{T} \quad \Delta \vdash \overline{\mathtt{V}} \preccurlyeq \overline{\mathtt{U}}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}'_i : \mathtt{V}'_i$
for some $\overline{\mathtt{V}}'$ such that $\Delta \vdash \mathtt{V}'_i \preccurlyeq \mathtt{V}_i$ and $\mathtt{V}'_j = \mathtt{V}_j$ for $j \neq i$. By Lemma 26, we have
$\Delta \vdash \overline{\mathtt{V}}' \preccurlyeq \overline{\mathtt{U}}$
Therefore, by the rule TR-INVK1, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{v}}, \mathtt{e}'_i, \overline{\mathtt{e}}) : \mathtt{T}$
Then, letting $\mathtt{T}' = \mathtt{T}$ finishes the case.

Case RC-INVK-RECV2.
$\mathtt{e} = \mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}](\overline{\mathtt{e}}) \quad \mathtt{e}' = \mathtt{e}'_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}](\overline{\mathtt{e}})$
$\mathtt{e}_0 \longrightarrow \mathtt{e}'_0$
By TR-INVK2, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N} \quad \Delta \vdash \mathtt{N} \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}$
$\mathit{mtype}(\mathtt{m}, \mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}) = \overline{\mathtt{U}} \to \mathtt{U} \quad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \Delta \vdash \overline{\mathtt{V}} \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \quad \mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}'_0 : \mathtt{T}'_0$
for some $\mathtt{T}'_0$ such that $\Delta \vdash \mathtt{T}'_0 \preccurlyeq \mathtt{T}_0$. By Lemma 37 and Lemma 26, we have
$\mathit{bound}_\Delta(\mathtt{T}'_0) = \mathtt{P} \quad \Delta \vdash \mathtt{P} \preccurlyeq \mathtt{N}$
$\Delta \vdash \mathtt{P} \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}$
Therefore, by the rule TR-INVK2, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}'_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}](\overline{\mathtt{e}}) : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
Then, letting $\mathtt{T}' = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$ finishes the case.

Case RC-INVK-ARG2.
$\mathtt{e} = \mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}](\overline{\mathtt{v}}, \mathtt{e}_i, \overline{\mathtt{e}}) \quad \mathtt{e}' = \mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}](\overline{\mathtt{v}}, \mathtt{e}'_i, \overline{\mathtt{e}})$
$\mathtt{e}_i \longrightarrow \mathtt{e}'_i$
By TR-INVK2, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N} \quad \Delta \vdash \mathtt{N} \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}$
$\mathit{mtype}(\mathtt{m}, \mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}) = \overline{\mathtt{U}} \to \mathtt{U} \quad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{v}}, \mathtt{e}_i, \overline{\mathtt{e}} : \overline{\mathtt{V}}$
$\Delta \vdash \overline{\mathtt{V}} \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \quad \mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}'_i : \mathtt{V}'_i$
for some $\overline{\mathtt{V}}'$ such that $\Delta \vdash \mathtt{V}'_i \preccurlyeq \mathtt{V}_i$ and $\mathtt{V}'_j = \mathtt{V}_j$ for $j \neq i$. By Lemma 26, we have
$\Delta \vdash \overline{\mathtt{V}}' \preccurlyeq [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}$
Therefore, by the rule TR-INVK2, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0.\mathtt{m}[\overline{\mathtt{U}}|\mathtt{C}\texttt{<}\overline{\mathtt{X}}\texttt{>}](\overline{\mathtt{v}}, \mathtt{e}'_i, \overline{\mathtt{e}}) : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
Then, letting $\mathtt{T}' = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$ finishes the case.

Case RC-INVK-RECV3.
$\mathtt{e} = \mathtt{invoke}(\mathtt{e}_0, \mathtt{m}, \overline{\mathtt{e}}) \quad \mathtt{e} = \mathtt{invoke}(\mathtt{e}'_0, \mathtt{m}, \overline{\mathtt{e}})$
$\mathtt{e}_0 \longrightarrow \mathtt{e}'_0$
By TR-INVK3, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{U}} \quad \mathtt{T} = \mathtt{dyn}\texttt{<}\mathtt{Object}\texttt{>}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}'_0 : \mathtt{T}'_0$
for some $\mathtt{T}'_0$. Therefore, by the rule TR-INVK3, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{invoke}(\mathtt{e}'_0, \mathtt{m}, \overline{\mathtt{e}}) : \mathtt{dyn}\texttt{<}\mathtt{Object}\texttt{>}$
Then, letting $\mathtt{T}' = \mathtt{dyn}\texttt{<}\mathtt{Object}\texttt{>}$ finishes the case.

Case RC-INVK-ARG3.
$\mathtt{e} = \mathtt{invoke}(\mathtt{e}_0, \mathtt{m}, \overline{\mathtt{v}}, \mathtt{e}_i, \overline{\mathtt{e}}) \quad \mathtt{e} = \mathtt{invoke}(\mathtt{e}_0, \mathtt{m}, \overline{\mathtt{v}}, \mathtt{e}_i, \overline{\mathtt{e}})$
$\mathtt{e}_i \longrightarrow \mathtt{e}'_i$
By TR-INVK3, we have
$\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{v}}, \mathtt{e}_i, \overline{\mathtt{e}} : \overline{\mathtt{U}} \quad \mathtt{T} = \mathtt{dyn}\texttt{<}\mathtt{Object}\texttt{>}$
By the induction hypothesis, we have

$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}'_i : \mathtt{U}'_i$
for some $\mathtt{U}'_i$. Therefore, by the rule TR-INVK3, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{invoke(e_0, m, \overline{v}, e_i, \overline{e}) : dyn\texttt{<}Object\texttt{>}}$
Then, letting $\mathtt{T}' = \mathtt{dyn\texttt{<}Object\texttt{>}}$ finishes the case.

Case RC-NEW-ARG.
$\mathtt{e} = \mathtt{new\ N(\overline{v}, e_i, \overline{e})} \quad \mathtt{e}' = \mathtt{new\ N(\overline{v}, e'_i, \overline{e})} \quad \mathtt{e}_i \longrightarrow \mathtt{e}'_i$
By TR-NEW, we have
$\Delta \vdash \mathtt{N\ ok} \quad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{\overline{v}, e_i, \overline{e} : \overline{U}} \quad \Delta \vdash \mathtt{\overline{U}} \preccurlyeq \mathtt{\overline{T}} \quad \mathtt{T} = \mathtt{N}$
By the induction hypothesis, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}'_i : \mathtt{U}'_i$
for some $\overline{\mathtt{U}}'$ such that $\Delta \vdash \mathtt{U}'_i \preccurlyeq \mathtt{U}_i$ and $\mathtt{U}'_j = \mathtt{U}_j$ for $j \neq i$. By Lemma 26, we have
$\Delta \vdash \overline{\mathtt{U}}' \preccurlyeq \overline{\mathtt{T}}$
Therefore, by the rule TR-NEW, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{new\ N(\overline{v}, e'_i, \overline{e}) : N}$
Then, letting $\mathtt{T}' = \mathtt{N}$ finishes the case.

Case RC-CAST.
$\mathtt{e} = (\!|\mathtt{T}|\!)\,\mathtt{e}_0 \quad \mathtt{e}' = (\!|\mathtt{T}|\!)\,\mathtt{e}'_0 \quad \mathtt{e}_0 \longrightarrow \mathtt{e}'_0$
By TR-CAST, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{S}$
By the induction hypothesis, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}'_0 : \mathtt{S}'$
for some $\mathtt{S}'$. Therefore, by the rule TR-CAST, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} (\!|\mathtt{T}|\!)\,\mathtt{e}'_0 : \mathtt{T}$
Then, letting $\mathtt{T}' = \mathtt{T}$ finishes the case.

Case E-*, EC-*.
$\mathtt{e}' = \mathtt{Error[}\mathcal{E}\mathtt{]}$
By TR-ERROR, we have
$\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{Error[}\mathcal{E}\mathtt{] : T}'$
and letting $\mathtt{T} = \mathtt{T}'$ finishes the case.

□

Theorem 52. Suppose e is a well-typed expression. If e is either

1. a field access $\mathtt{e_0.f}$,
2. a field access with run-time check $\mathtt{get(e_0, f)}$,
3. a method invocation $\mathtt{e_0.m(\overline{e})}$,
4. a method invocation with argument check $\mathtt{e_0.m[\overline{U}|C\texttt{<}\overline{X}\texttt{>}](\overline{e})}$
5. a method invocation with run-time check $\mathtt{invoke(e_0, m, \overline{e})}$, or
6. a cast $(\!|\mathtt{S}|\!)\,\mathtt{e_0}$,

then there exist some $\mathtt{e}'$ such that $\mathtt{e} \longrightarrow \mathtt{e}'$.

Proof. By induction on the derivation of $\Delta; \Gamma \vdash_{\mathrm{G}} \mathtt{e : T}$ with a case analysis on the last rule used.

Case TR-FIELD1.
$\mathtt{e} = \mathtt{e_0.f}_i \quad \Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$
$\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \quad \mathtt{T} = \mathtt{T}_i$
If $\mathtt{e}_0$ is not a value, then by the induction hypothesis, we have
$\mathtt{e}_0 \longrightarrow \mathtt{e}'_0$
for some $\mathtt{e}'_0$ and applying RC-FIELD1 or EC-FIELD1 finishes the case. Otherwise, if $\mathtt{e}'_0 = \mathtt{new\ N(\overline{v})}$, then applying R-FIELD1 finishes the case.

Case TR-FIELD2.
$\mathtt{e} = \mathtt{get(e_0, f)} \quad \Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \mathtt{T} = \mathtt{dyn\texttt{<}Object\texttt{>}}$
If $\mathtt{e}_0$ is not a value, then by the induction hypothesis, we have

$\mathtt{e}_0 \longrightarrow \mathtt{e}'_0$

for some $\mathtt{e}'_0$ and applying RC-FIELD2 or EC-FIELD2 finishes the case. Otherwise, if $\mathtt{e}'_0 = \mathtt{new\ N(\overline{v})}$, then we have

$\mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$

Then, applying R-FIELD2 (when $\mathtt{f} \in \overline{\mathtt{f}}$) or E-FIELD (when $\mathtt{f} \notin \overline{\mathtt{f}}$) finishes the case.

Case TR-INVK1.

$\mathtt{e} = \mathtt{e}_0\mathtt{.m(}\overline{\mathtt{e}}\mathtt{)} \quad \Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0 \quad \Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{Q} \quad \Delta \vdash \mathtt{Q} \nsim \mathtt{N}$

$\mathit{dynfree}_\Delta(\mathtt{N}) \quad \mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{T} \quad \Delta \vdash \overline{\mathtt{V}} \nsim \overline{\mathtt{S}}$

If $\mathtt{e}_0$ is not a value, then by the induction hypothesis, we have

$\mathtt{e}_0 \longrightarrow \mathtt{e}'_0$

for some $\mathtt{e}'_0$ and applying RC-INVK-RECV1 or EC-INVK-RECV1 finishes the case. If $\mathtt{e}_i$ is not a value, then by the induction hypothesis, we have

$\mathtt{e}_i \longrightarrow \mathtt{e}'_i$

for some $\mathtt{e}'_i$ and applying RC-INVK-ARG1 or EC-INVK-ARG1 finishes the case. Otherwise, if $\mathtt{e}_0 = \mathtt{new\ Q(\overline{v})}$ and $\overline{\mathtt{e}} = \mathtt{new}\ \overline{\mathtt{P}}\mathtt{(...)}$, then by definition of mtype and mbody and the rule TR-CLASS, we have

$\mathit{mbody}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{x}}\mathtt{.}\,\mathtt{e}'_0$

and applying R-INVK1 finishes the case.

Case TR-INVK2.

$\mathtt{e} = \mathtt{e}_0\mathtt{.m[}\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{Y}}\mathtt{>]\,(}\overline{\mathtt{e}}\mathtt{)} \quad \Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$

$\Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N} \quad \Delta \vdash \mathtt{N} \nsim [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{C<}\overline{\mathtt{Y}}\mathtt{>}$

$\mathit{mtype}(\mathtt{m}, \mathtt{C<}\overline{\mathtt{Y}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U} \quad \Delta \vdash \overline{\mathtt{V}} \nsim [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\overline{\mathtt{U}} \quad \mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{Y}}]\mathtt{U}$

If $\mathtt{e}_0$ is not a value, then by the induction hypothesis, we have

$\mathtt{e}_0 \longrightarrow \mathtt{e}'_0$

for some $\mathtt{e}'_0$ and applying RC-INVK-RECV2 or EC-INVK-RECV2 finishes the case. If $\mathtt{e}_i$ is not a value, then by the induction hypothesis, we have

$\mathtt{e}_i \longrightarrow \mathtt{e}'_i$

for some $\mathtt{e}'_i$ and applying RC-INVK-ARG2 or EC-INVK-ARG2 finishes the case. Otherwise, if $\mathtt{e}_0 = \mathtt{new\ N(\overline{v})}$ and $\overline{\mathtt{e}} = \mathtt{new}\ \overline{\mathtt{P}}\mathtt{(...)}$, then by definition of mtype and mbody and the rule TR-CLASS, we have

$\mathit{mbody}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{x}}\mathtt{.}\,\mathtt{e}'_0$

and applying R-INVK2 (when $\bullet \vdash \overline{\mathtt{P}} \nsim [\mathit{tyargs}(\mathtt{N}, \mathtt{C})/\overline{\mathtt{Y}}]\overline{\mathtt{U}}$) or E-INVK-ARG1 (when $\bullet \vdash \overline{\mathtt{P}} \not\sim [\mathit{tyargs}(\mathtt{N}, \mathtt{C})/\overline{\mathtt{Y}}]\overline{\mathtt{U}}$) finishes the case.

Case TR-INVK3.

$\mathtt{e} = \mathtt{invoke(e_0\,,\,m\,,}\,\overline{\mathtt{e}}\mathtt{)} \quad \Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{T}_0$

$\Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathtt{T} = \mathtt{dyn<Object>}$

If $\mathtt{e}_0$ is not a value, then by the induction hypothesis, we have

$\mathtt{e}_0 \longrightarrow \mathtt{e}'_0$

for some $\mathtt{e}'_0$ and applying RC-INVK-RECV3 or EC-INVK-RECV3 finishes the case. If $\mathtt{e}_i$ is not a value, then by the induction hypothesis, we have

$\mathtt{e}_i \longrightarrow \mathtt{e}'_i$

for some $\mathtt{e}'_i$ and applying RC-INVK-ARG3 or EC-INVK-ARG3 finishes the case. Otherwise, if $\mathtt{e}_0 = \mathtt{new\ N(\overline{v})}$ and $\overline{\mathtt{e}} = \mathtt{new}\ \overline{\mathtt{P}}\mathtt{(...)}$, then we have either $\mathit{nomethod}(\mathtt{m}, \mathtt{N})$ or $\mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{U}} \to \mathtt{U}$. In former case, applying E-INVK finishes the case. In latter case, by definition of mtype and mbody and the rule TR-CLASS, we have

$\mathit{mbody}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{x}}\mathtt{.}\,\mathtt{e}'_0$

and applying R-INVK3 (when $\bullet \vdash \overline{\mathtt{P}} \nsim \overline{\mathtt{U}}$) or E-INVK-ARG2 (when $\bullet \vdash \overline{\mathtt{P}} \not\sim \overline{\mathtt{U}}$) finishes the case.

Case TR-CAST.

$\mathtt{e} = (\!|\mathtt{T}|\!)\,\mathtt{e}_0 \quad \Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}_0 : \mathtt{V}$

If $\mathtt{e}_0$ is not a value, then by the induction hypothesis, we have

$\mathtt{e}_0 \longrightarrow \mathtt{e}'_0$

for some $\mathtt{e}'_0$ and applying RC-CAST or EC-CAST finishes the case. Otherwise, if $\mathtt{e}_0 = \mathtt{new\ N(\overline{v})}$, then applying R-CAST (when $\bullet \vdash \mathtt{N} \nsim \mathtt{T}$) or E-CAST (when $\bullet \vdash \mathtt{N} \not\sim \mathtt{T}$).

□

B.3.1 Proof of Theorem 8

Proof. Immediate from Theorem 51 and Theorem 52 □

B.3.2 Proof of Theorem 9

Proof. Immediate from subject reduction and progress properties and the fact that there is no rule reducing a non-run-time-check expression to an errror. □

B.4 Translation

Lemma 53. $\Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e} : \mathtt{T}$ iff $\exists \mathtt{e}'.\ \Delta;\Gamma \vdash \mathtt{e} \rightsquigarrow \mathtt{e}' : \mathtt{T}$.

Proof. ($\Rightarrow$) By induction on the derivation of $\Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e} : \mathtt{T}$ with a case analysis on the last rule used.

Case TG-VAR.
$\mathtt{e} = \mathtt{x} \quad \mathtt{T} = \Gamma(\mathtt{x})$
By the rule TRNS-VAR, we have
$\Delta;\Gamma \vdash \mathtt{x} \rightsquigarrow \mathtt{x} : \Gamma(\mathtt{x})$
and letting $\mathtt{e}' = \mathtt{x}$ finishes the case.

Case TG-FIELD1.
$\mathtt{e} = \mathtt{e}_0.\mathtt{f}_i \quad \Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0 : \mathtt{T}_0$
$\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \quad \mathtt{T} = \mathtt{T}_i$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{T}_0$
for some $\mathtt{e}'_0$. Then, by the rule TRNS-FIELD1, we have
$\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{f}_i \rightsquigarrow \mathtt{e}'_0.\mathtt{f}_i : \mathtt{T}_i$
and letting $\mathtt{e}' = \mathtt{e}'_0.\mathtt{f}_i$ finishes the case.

Case TG-FIELD2.
$\mathtt{e} = \mathtt{e}_0.\mathtt{f} \quad \Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0 : \mathtt{dyn\texttt{<}N\texttt{>}}$
$\mathit{fields}(\mathit{bound}_\Delta(\mathtt{dyn\texttt{<}N\texttt{>}})) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \quad \mathtt{f} \notin \overline{\mathtt{f}}$
$\mathtt{T} = \mathtt{dyn\texttt{<}Object\texttt{>}}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{dyn\texttt{<}N\texttt{>}}$
for some $\mathtt{e}'_0$. Then, by the rule TRNS-FIELD2, we have
$\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{f} \rightsquigarrow \mathtt{get(e}'_0\mathtt{, f)} : \mathtt{dyn\texttt{<}Object\texttt{>}}$
and letting $\mathtt{e}' = \mathtt{get(e}'_0\mathtt{, f)}$ finishes the case.

Case TG-INVK1.
$\mathtt{e} = \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \quad \Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0 : \mathtt{T}_0$
$\Delta;\Gamma \vdash_{\mathrm{G}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C\texttt{<}\overline{X}\texttt{>}}$
$\mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{T} \quad \Delta \vdash \overline{\mathtt{V}} \lesssim \overline{\mathtt{S}}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{T}_0$
$\Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{V}}$
for some $\mathtt{e}'_0$ and $\overline{\mathtt{e}}'$. By the definition of mtype, we have
$\mathit{mtype}(\mathtt{m}, \mathtt{C\texttt{<}\overline{X}\texttt{>}}) = \overline{\mathtt{U}} \to \mathtt{U}$
$\overline{\mathtt{S}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \quad \mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
for some $\overline{\mathtt{U}}$ and $\mathtt{U}$. If $\mathit{dynfree}_\Delta(\mathtt{N})$, then by the rule TRNS-INVK1, we have
$\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \rightsquigarrow \mathtt{e}'_0.\mathtt{m}(\overline{\mathtt{e}}') : \mathtt{T}$
and letting $\mathtt{e}' = \mathtt{e}'_0.\mathtt{m}(\overline{\mathtt{e}}')$ finishes the case. Otherwise, by the rule TRNS-INVK2, we have
$\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \rightsquigarrow \mathtt{e}_0.\mathtt{m[\overline{U}|C\texttt{<}\overline{X}\texttt{>}]}(\langle[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \Leftarrow \overline{\mathtt{V}}\rangle_\Delta \overline{\mathtt{e}}) : [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
and letting $\mathtt{e}' = \mathtt{e}_0.\mathtt{m[\overline{U}|C\texttt{<}\overline{X}\texttt{>}]}(\langle[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \Leftarrow \overline{\mathtt{V}}\rangle_\Delta \overline{\mathtt{e}})$ finishes the case.

Case TG-INVK2.
$\mathtt{e} = \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \quad \Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0 : \mathtt{dyn\texttt{<}N\texttt{>}}$
$\mathit{nomethod}(\mathtt{m}, \mathit{bound}_\Delta(\mathtt{dyn\texttt{<}N\texttt{>}}))$
$\Delta;\Gamma \vdash_{\mathrm{G}} \overline{\mathtt{e}} : \overline{\mathtt{V}} \quad \mathtt{T} = \mathtt{dyn\texttt{<}Object\texttt{>}}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{T}_0$
$\Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{V}}$
Then, by the rule TRNS-INVK3, we have

$\Delta;\Gamma \vdash \mathtt{e}_0\mathtt{.m}(\overline{\mathtt{e}}) \rightsquigarrow \mathtt{invoke}(\mathtt{e}'_0, \mathtt{m}, \overline{\mathtt{e}}')\mathtt{:dyn<Object>}$
and letting $\mathtt{e}' = \mathtt{invoke}(\mathtt{e}'_0, \mathtt{m}, \overline{\mathtt{e}}')$ finishes the case.

Case TG-NEW.
$\mathtt{e} = \mathtt{new\ N}(\overline{\mathtt{e}})$ $\Delta \vdash \mathtt{N}$ ok
$\mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$ $\Delta;\Gamma \vdash_{\mathrm{G}} \overline{\mathtt{e}}:\overline{\mathtt{U}}$
$\Delta \vdash \overline{\mathtt{U}} \lesssim \overline{\mathtt{T}}$ $\mathtt{T} = \mathtt{N}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}':\overline{\mathtt{U}}$
Then, by the rule TRNS-NEW, we have
$\Delta;\Gamma \vdash \mathtt{new\ N}(\overline{\mathtt{e}}) \rightsquigarrow \mathtt{new\ N}(\langle\overline{\mathtt{T}} \Leftarrow \overline{\mathtt{U}}\rangle_\Delta\overline{\mathtt{e}}')\mathtt{:N}$
and letting $\mathtt{e}' = \mathtt{new\ N}(\langle\overline{\mathtt{T}} \Leftarrow \overline{\mathtt{U}}\rangle_\Delta\overline{\mathtt{e}}')$ finishes the case.

($\Leftarrow$) By induction on the derivation of $\Delta;\Gamma \vdash \mathtt{e} \rightsquigarrow \mathtt{e}':\mathtt{T}$ with a case analysis on the last rule used.

Case TRNS-VAR.
$\mathtt{e} = \mathtt{x}$ $\mathtt{e}' = \mathtt{x}$ $\mathtt{T} = \Gamma(\mathtt{x})$
The, conclusion is immediate from the rule TG-VAR.

Case TRNS-FIELD1.
$\mathtt{e} = \mathtt{e}_0\mathtt{.f}_i$ $\mathtt{e}' = \mathtt{e}'_0\mathtt{.f}_i$ $\mathtt{T} = \mathtt{T}_i$
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0\mathtt{:T}_0$ $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0\mathtt{:T}_0$
Then, the conclusion is immediate from the rule TG-FIELD1.

Case TRNS-FIELD2.
$\mathtt{e} = \mathtt{e}_0\mathtt{.f}$ $\mathtt{e}' = \mathtt{get}(\mathtt{e}'_0\mathtt{, f})$ $\mathtt{T} = \mathtt{dyn<Object>}$
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0\mathtt{:dyn<N>}$ $\mathit{fields}(\mathit{bound}_\Delta(\mathtt{dyn<N>})) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$ $\mathtt{f} \notin \overline{\mathtt{f}}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0\mathtt{:dyn<N>}$
Then, the conclusion is immediate from the rule TG-FIELD2.

Case TRNS-INVK1.
$\mathtt{e} = \mathtt{e}_0\mathtt{.m}(\overline{\mathtt{e}})$ $\mathtt{e}' = \mathtt{e}'_0\mathtt{.m}(\langle\overline{\mathtt{S}} \Leftarrow \overline{\mathtt{V}}\rangle_\Delta\overline{\mathtt{e}}')$
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0\mathtt{:T}_0$ $\Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}':\overline{\mathtt{V}}$
$\mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N}$ $\mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{S}$ $\Delta \vdash \overline{\mathtt{V}} \lesssim \overline{\mathtt{S}}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0\mathtt{:T}_0$ $\Delta;\Gamma \vdash_{\mathrm{G}} \overline{\mathtt{e}}:\overline{\mathtt{V}}$
Then, the conclusion is immediate from the rule TG-INVK1.

Case TRNS-INVK2.
$\mathtt{e} = \mathtt{e}_0\mathtt{.m}[\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>}](\overline{\mathtt{e}})$ $\mathtt{e}' = \mathtt{e}'_0\mathtt{.m}[\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>}](\langle[\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \Leftarrow \overline{\mathtt{V}}\rangle_\Delta\overline{\mathtt{e}}')$ $\mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0\mathtt{:T}_0$ $\Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}':\overline{\mathtt{V}}$
$\mathit{bound}_\Delta(\mathtt{T}_0) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<}\overline{\mathtt{X}}\mathtt{>}$ $\mathit{mtype}(\mathtt{m}, \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U}$
$\Delta \vdash \overline{\mathtt{V}} \lesssim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0\mathtt{:T}_0$ $\Delta;\Gamma \vdash_{\mathrm{G}} \overline{\mathtt{e}}:\overline{\mathtt{V}}$
By the definition of mtype, we have
$\mathit{mtype}(\mathtt{m}, \mathit{bound}_\Delta(\mathtt{T}_0)) = \mathit{mtype}(\mathtt{m}, \mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) = \overline{\mathtt{S}} \to \mathtt{S}$
$\overline{\mathtt{S}} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}$ $\mathtt{S} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
Then, the conclusion is immediate from the rule TG-INVK1.

Case TRNS-INVK3.
$\mathtt{e} = \mathtt{e}_0\mathtt{.m}(\overline{\mathtt{e}})$ $\mathtt{e}' = \mathtt{invoke}(\mathtt{e}'_0\mathtt{, m, }\overline{\mathtt{e}}')$ $\mathtt{T} = \mathtt{dyn<Object>}$
$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0\mathtt{:dyn<N>}$ $\Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}':\overline{\mathtt{V}}$
$\mathit{nomethod}(\mathtt{m}, \mathit{bound}_\Delta(\mathtt{dyn<N>}))$
By the induction hypothesis, we have
$\Delta;\Gamma \vdash_{\mathrm{G}} \mathtt{e}_0\mathtt{:dyn<N>}$ $\Delta;\Gamma \vdash_{\mathrm{G}} \overline{\mathtt{e}}:\overline{\mathtt{V}}$
Then, the conclusion is immediate from the rule TG-INVK2.

Case TRNS-NEW.

$\mathtt{e} = \mathtt{new\ N(\overline{e})} \quad \mathtt{e'} = \mathtt{new\ N}(\langle \overline{\mathtt{T}} \Leftarrow \overline{\mathtt{U}} \rangle_\Delta \overline{\mathtt{e}}') \quad \mathtt{T} = \mathtt{N}$
$\Delta \vdash \mathtt{N\ ok} \quad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$
$\Delta; \Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{U}} \quad \Delta \vdash \overline{\mathtt{U}} \lesssim \overline{\mathtt{T}}$
By the induction hypothesis, we have
$\Delta; \Gamma \vdash_{\mathrm{G}} \overline{\mathtt{e}} : \overline{\mathtt{U}}$
Then, the conclusion is immediate from the rule TG-NEW.

□

Lemma 54. If CT OK IN FGJ$^{\mathtt{dyn}}$, then $CT \rightsquigarrow CT'$ and $CT' \in$ FGJ$^{(\!(\ )\!)}$ for some CT'.

Proof. Let $CT(\mathtt{C}) = \mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\ \mathtt{K}\ \overline{\mathtt{M}}\}$. By CT OK IN FGJ$^{\mathtt{dyn}}$, we have
$\forall \mathtt{N}_i \in \overline{\mathtt{N}}.(\mathtt{X}_1^{\kappa_1} <: \mathtt{N}_1, \ldots, \mathtt{X}_{i-1}^{\kappa_{i-1}} <: \mathtt{N}_{i-1} \vdash \mathtt{N}_i\ \mathrm{ok})$
$\overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}} \vdash \mathtt{N}, \overline{\mathtt{T}}\ \mathrm{ok} \quad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{U}}\ \overline{\mathtt{g}} \quad \overline{\mathtt{M}}\ \mathtt{OK\ IN\ C<}\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\mathtt{>}$
$\mathtt{K} = \ \mathtt{C(}\overline{\mathtt{U}}\ \overline{\mathtt{g}}\mathtt{,}\ \overline{\mathtt{T}}\ \overline{\mathtt{f}}\mathtt{)\{super(}\overline{\mathtt{g}}\mathtt{);\ this.}\overline{\mathtt{f}}\mathtt{=}\overline{\mathtt{f}}\mathtt{;\}}$
Then, for each $\mathtt{M}_j \in \overline{\mathtt{M}}$, we have
$\mathtt{M}_j = \ \mathtt{V\ m(}\overline{\mathtt{V}}\ \overline{\mathtt{x}}\mathtt{)\{\ return\ e_0;\ \}} \quad \Delta = \overline{\mathtt{X}}^{\overline{\kappa}} <: \overline{\mathtt{N}}$
$\Delta \vdash \overline{\mathtt{V}}, \mathtt{V}\ \mathrm{ok} \quad \Delta; \overline{\mathtt{x}} : \overline{\mathtt{V}}, \mathtt{this} : \mathtt{C<}\overline{\mathtt{X}}\mathtt{>} \vdash_{\mathrm{G}} \mathtt{e_0} : \mathtt{S}$
$\Delta \vdash \mathtt{S} \lesssim \mathtt{V} \quad \mathit{override}_\Delta(\mathtt{m}, \mathtt{N}, \overline{\mathtt{V}} \to \mathtt{V})$
By Lemma 53, we have
$\Delta; \overline{\mathtt{x}} : \overline{\mathtt{V}}, \mathtt{this} : \mathtt{C<}\overline{\mathtt{X}}\mathtt{>} \vdash \mathtt{e_0} \rightsquigarrow \mathtt{e}_0' : \mathtt{S}$
Then, by the rule TRNS-METHOD, we have
$\mathtt{M}_j' = \ \mathtt{V\ m(}\overline{\mathtt{V}}\ \overline{\mathtt{x}}\mathtt{)\{\ return\ } \langle \mathtt{V} \Leftarrow \mathtt{S} \rangle_\Delta \mathtt{e}_0'\mathtt{;\ \}}$
$\mathtt{M}_j \rightsquigarrow \mathtt{M}_j'\ \mathtt{IN\ C<}\overline{\mathtt{X}} \lhd \overline{\mathtt{N}}\mathtt{>}$
Finally, by the rule TRNS-CLASS, we have
$\mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\ \mathtt{K}\ \overline{\mathtt{M}}\} \rightsquigarrow \mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\ \mathtt{K}\ \overline{\mathtt{M}}'\}$
and letting $CT'(\mathtt{C}) = \mathtt{class\ C<}\overline{\mathtt{X}}^{\overline{\kappa}} \lhd \overline{\mathtt{N}}\mathtt{>} \lhd \mathtt{N}\ \{\overline{\mathtt{T}}\ \overline{\mathtt{f}};\ \mathtt{K}\ \overline{\mathtt{M}}'\}$ finishes the proof. □

Lemma 55. If $\Delta; \Gamma \vdash_{\mathrm{G}} \mathtt{e} : \mathtt{T}$ under CT OK IN FGJ$^{\mathtt{dyn}}$ and $CT \rightsquigarrow CT'$, then $\Delta; \Gamma \vdash \mathtt{e} \rightsquigarrow \mathtt{e}' : \mathtt{T}$ for some $\mathtt{e}'$ and $\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}' : \mathtt{T}$ under CT'.

Proof. By Lemma 53, we have some $\mathtt{e}'$ such that $\Delta; \Gamma \vdash \mathtt{e} \rightsquigarrow \mathtt{e}' : \mathtt{T}$. By induction on the derivation of $\Delta; \Gamma \vdash \mathtt{e} \rightsquigarrow \mathtt{e}' : \mathtt{T}$ with a case analysis on the last rule used.

Case TRNS-VAR.
$\mathtt{e} = \mathtt{x} \quad \mathtt{e'} = \mathtt{x} \quad \mathtt{T} = \Gamma(\mathtt{x})$
The conclusion is immediate from the rule TR-VAR.

Case TRNS-FIELD1.
$\mathtt{e} = \mathtt{e_0.f} \quad \mathtt{e'} = \mathtt{e}_0'\mathtt{.f} \quad \mathtt{T} = \mathtt{T}_i$
$\Delta; \Gamma \vdash \mathtt{e_0} \rightsquigarrow \mathtt{e}_0' : \mathtt{T}_0 \quad \mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$
By the induction hypothesis, we have $\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0' : \mathtt{T}_0$. Then the conclusion is immediate from the rule TR-FIELD1.

Case TRNS-FIELD2.
$\mathtt{e} = \mathtt{e_0.f} \quad \mathtt{e'} = \mathtt{get(e}_0'\mathtt{,\ f)} \quad \mathtt{T} = \mathtt{dyn<Object>}$
$\Delta; \Gamma \vdash \mathtt{e_0} \rightsquigarrow \mathtt{e}_0' : \mathtt{dyn<N>} \quad \mathtt{f} \notin \overline{\mathtt{f}} \quad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}}$
By the induction hypothesis, we have $\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0' : \mathtt{dyn<N>}$. Then the conclusion is immediate from the rule TR-FIELD2.

Case TRNS-INVK1.
$\mathtt{e} = \mathtt{e_0.m(}\overline{\mathtt{e}}\mathtt{)} \quad \mathtt{e'} = \mathtt{e}_0'\mathtt{.m}(\langle \overline{\mathtt{S}} \Leftarrow \overline{\mathtt{V}} \rangle_\Delta \overline{\mathtt{e}}')$
$\Delta; \Gamma \vdash \mathtt{e_0} \rightsquigarrow \mathtt{e}_0' : \mathtt{T}_0 \quad \Delta; \Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{V}} \quad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{N}$
$\mathit{dynfree}_\Delta(\mathtt{N}) \quad \mathit{mtype}(\mathtt{m}, \mathtt{N}) = \overline{\mathtt{S}} \to \mathtt{T} \quad \Delta \vdash \overline{\mathtt{V}} \lesssim \overline{\mathtt{S}}$
By the induction hypothesis, we have $\Delta; \Gamma \vdash_{\mathrm{R}} \mathtt{e}_0' : \mathtt{T}_0$ and $\Delta; \Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}}' : \overline{\mathtt{V}}$. By the rule TR-CAST, we have $\Delta; \Gamma \vdash_{\mathrm{R}} \langle \overline{\mathtt{S}} \Leftarrow \overline{\mathtt{V}} \rangle_\Delta \overline{\mathtt{e}}' : \overline{\mathtt{V}}'$ for some $\overline{\mathtt{V}}'$ such that $\Delta \vdash \overline{\mathtt{V}}' \nprec \overline{\mathtt{S}}$. Then, applying the rule TR-INVK1 finishes the case.

Case TRNS-INVK2.
$\mathtt{e} = \mathtt{e_0.m(}\overline{\mathtt{e}}\mathtt{)} \quad \mathtt{e'} = \mathtt{e}_0'\mathtt{.m[}\overline{\mathtt{U}}\mathtt{|C<}\overline{\mathtt{X}}\mathtt{>]}(\langle [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \Leftarrow \overline{\mathtt{V}} \rangle_\Delta\ \overline{\mathtt{e}}') \quad \mathtt{T} = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{U}$
$\Delta; \Gamma \vdash \mathtt{e_0} \rightsquigarrow \mathtt{e}_0' : \mathtt{T}_0 \quad \Delta; \Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{V}} \quad \mathit{bound}_\Delta(\mathtt{T}_0) = [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\mathtt{C<}\overline{\mathtt{X}}\mathtt{>}$
$\neg \mathit{dynfree}_\Delta(\mathtt{C<}\overline{\mathtt{T}}\mathtt{>}) \quad \mathit{mtype}(\mathtt{m}, \mathtt{C<}\overline{\mathtt{X}}\mathtt{>}) = \overline{\mathtt{U}} \to \mathtt{U} \quad \Delta \vdash \overline{\mathtt{V}} \lesssim [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}$

By the induction hypothesis, we have $\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}'_0 : \mathtt{T}_0$ and $\Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}}' : \overline{\mathtt{V}}$. By the rule TR-CAST, we have $\Delta;\Gamma \vdash_{\mathrm{R}} \langle [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}} \Leftarrow \overline{\mathtt{V}} \rangle_{\Delta} \overline{\mathtt{e}}' : \overline{\mathtt{V}}'$ for some $\overline{\mathtt{V}}'$ such that $\Delta \vdash \overline{\mathtt{V}}' \nprec [\overline{\mathtt{T}}/\overline{\mathtt{X}}]\overline{\mathtt{U}}$. Then, applying the rule TR-INVK2 finishes the case.

Case TRNS-INVK3.

$\mathtt{e} = \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) \quad \mathtt{e}' = \mathtt{invoke}(\mathtt{e}'_0, \mathtt{m}, \overline{\mathtt{e}}') \quad \mathtt{T} = \mathtt{dyn\texttt{<}Object\texttt{>}}$

$\Delta;\Gamma \vdash \mathtt{e}_0 \rightsquigarrow \mathtt{e}'_0 : \mathtt{dyn\texttt{<}N\texttt{>}} \quad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{V}} \quad \mathit{nomethod}(\mathtt{m}, \mathtt{N})$

By the induction hypothesis, we have $\Delta;\Gamma \vdash_{\mathrm{R}} \mathtt{e}'_0 : \mathtt{dyn\texttt{<}N\texttt{>}}$ and $\Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}}' : \overline{\mathtt{V}}$. Then, applying the rule TR-INVK3 finishes the case.

Case TRNS-NEW.

$\mathtt{e} = \mathtt{new\ N}(\overline{\mathtt{e}}) \quad \mathtt{e}' = \mathtt{new\ N}(\langle \overline{\mathtt{T}} \Leftarrow \overline{\mathtt{U}} \rangle_{\Delta} \overline{\mathtt{e}}') \quad \mathtt{T} = \mathtt{N}$

$\Delta \vdash \mathtt{N}\ \mathrm{ok} \quad \mathit{fields}(\mathtt{N}) = \overline{\mathtt{T}}\ \overline{\mathtt{f}} \quad \Delta;\Gamma \vdash \overline{\mathtt{e}} \rightsquigarrow \overline{\mathtt{e}}' : \overline{\mathtt{U}} \quad \Delta \vdash \overline{\mathtt{U}} \lesssim \overline{\mathtt{T}}$

By the induction hypothesis, we have $\Delta;\Gamma \vdash_{\mathrm{R}} \overline{\mathtt{e}}' : \overline{\mathtt{U}}$. By the rule TR-CAST, we have $\Delta;\Gamma \vdash_{\mathrm{R}} \langle \overline{\mathtt{T}} \Leftarrow \overline{\mathtt{U}} \rangle_{\Delta} \overline{\mathtt{e}}' : \overline{\mathtt{U}}'$ for some $\overline{\mathtt{U}}'$ such that $\Delta \vdash \overline{\mathtt{U}}' \nprec \overline{\mathtt{T}}$. Then, applying the rule TR-NEW finishes the case.

□

B.4.1 Proof of Theorem 10

Proof. By and Lemma 54, we have a translated program $(CT', \mathtt{e}')$. By Lemma 55, we have $\bullet;\bullet \vdash_{\mathrm{R}} \mathtt{e}' : \mathtt{T}$ and all expressions in CT' are well typed, i.e., $(CT', \mathtt{e}')$ is a well-formed program. □

B.4.2 Proof of Theorem 11

Proof. By Theorem 10, we have well-typed program $(CT', \mathtt{e}')$. By the translation rules and Lemma 5, there is no run-time check in $(CT', \mathtt{e}')$. □