# 蕁　絅膣荐膊
# ≪　c 鴻ゅ

√臼 紊　鐚篋綉 羞

篋　遵ぇ絖　ぇ絖　∽怨　腥句

April 16, 2012

# 絅膣

- 膣荅括完　罕
- 　腥吟 c　違　羣鴻＞散
  - 　違　絎茵
  - ∽違　＞散私緇＞散鐚箏紊＞散

Eiffel, Racket, D,...

# 絅膣荐膊 [1]

## 蕭∽違　　絅膣≪　荐膊篏膺

**蕁　(manifest) 絅膣荐膊**

**膣宴障**

$\{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} > 0\}$

**篋**: $\lambda^{H}$[2]

**羮　(latent) 絅膣荐膊**

**膣宴障**

$\mathtt{nat}$

**篋**: $\lambda^{\textsc{Con}}$[1]


[1] Findler and Felleisen, 2002

[2] Knowles and Flanagan, 2010

# 絅膣荐膊 [1]

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$\ni 1, 2, 3, \ldots$


[1] Findler and Felleisen, 2002

[2] Knowles and Flanagan, 2010

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$\not\ni 0$


[1] Findler and Felleisen, 2002
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# 絅膣荐膊 [1]

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**膣宴障**

$\mathtt{nat}$

**篋**: $\lambda^{\text{Con}}$[1]

$\ni 1, 2, 3, \ldots$

$\not\ni 0$

[1] Findler and Felleisen, 2002
[2] Knowles and Flanagan, 2010

c 鴻 $\langle T_1 \Rightarrow T_2 \rangle^{\ell}$ 絅膣 祉

- $T_1$ や $T_2$ 絎茵鑺
- 鑺祉 け遺紊榊

$$\langle \mathtt{nat} \Rightarrow \{x : \mathtt{nat} \mid x > 0\} \rangle^{\ell}\ 2 \ \rightsquigarrow^{*}\ 2$$

$$\langle \mathtt{nat} \Rightarrow \{x : \mathtt{nat} \mid x > 0\} \rangle^{\ell}\ 0 \ \rightsquigarrow^{*}\ \Uparrow \ell$$

# c 鴻　蕁: 荐膊潟鴻紜紊

- c 鴻　贋　荀
  - 篁
- 荅箴＜　襲帥　c 鴻吾綽荀
  - 綽　c 鴻

$$\begin{array}{lll} \mathtt{div} & : & \mathtt{nat} \rightarrow \{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} \neq \mathtt{0}\} \rightarrow \mathtt{nat} \\ \mathtt{succ} & : & \mathtt{nat} \rightarrow \{\mathtt{y} : \mathtt{nat} \mid \mathtt{y} > \mathtt{0}\} \end{array}$$

$$\nvdash \mathtt{div}\ \mathtt{5}\ (\mathtt{succ}\ \mathtt{0}) : \mathtt{nat}$$

## c 鴻 蕁: 荐膊潟鴻紜紊

- c 鴻 贋 荀
  - 篁
- 荅箴< 襲帥 c 鴻吾綽荀
  - 綽 c 鴻

$$
\begin{aligned}
\texttt{div} &: \texttt{nat} \to \{x : \texttt{nat} \mid x \neq 0\} \to \texttt{nat} \\
\texttt{succ} &: \texttt{nat} \to \{y : \texttt{nat} \mid y > 0\}
\end{aligned}
$$

$$\nvdash \texttt{div}\ 5\ \underline{(\texttt{succ}\ 0)} : \texttt{nat}$$

- c鴻 贋 荀
  - 篁
- 荅箴＜ 襲帥 c鴻吾綽荀
  - 綽 c鴻

$$
\begin{aligned}
\mathtt{div} &: \mathtt{nat} \to \{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} \neq 0\} \to \mathtt{nat} \\
\mathtt{succ} &: \mathtt{nat} \to \{\mathtt{y} : \mathtt{nat} \mid \mathtt{y} > 0\}
\end{aligned}
$$

$$\nvdash \ \mathtt{div}\ 5\ \underline{(\mathtt{succ}\ 0)} : \mathtt{nat}$$

$$\vdash \ \mathtt{div}\ 5\ \underline{(\langle\{\mathtt{y} : \mathtt{nat} \mid \mathtt{y} > 0\} \Rightarrow \{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} \neq 0\}\rangle^{\ell}\ (\mathtt{succ}\ 0))} : \mathtt{nat}$$

# c 鴻　蕁: 荐膊潟鴻紜系

**篁　箴**

$$\texttt{div 5}$$
$$(\langle \{y : \texttt{nat} \mid y > 0\} \Rightarrow \{x : \texttt{nat} \mid x \neq 0\}\rangle^{\ell}\ \texttt{(succ 0)}) \rightsquigarrow$$

**篁 c　箴**

`div 5 (succ 0)`

# c 鴻 蕁: 荐膊潟鴻絋紊

**篁 箴**

$\texttt{div 5}$

$(\langle\{\mathtt{y} : \mathtt{nat} \mid \mathtt{y} > 0\} \Rightarrow \{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} \neq 0\}\rangle^{\ell} \ (\underline{\mathtt{succ\ \ 0}})) \ \rightsquigarrow$

**篁 c 箴**

$\texttt{div 5 (succ 0)}$

**篁　箴**

div 5
$(\langle\{\mathtt{y} : \mathtt{nat} \mid \mathtt{y} > 0\} \Rightarrow \{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} \neq 0\}\rangle^{\ell}\ (\underline{\mathtt{succ}\ 0})) \rightsquigarrow$

div 5
$(\langle\{\mathtt{y} : \mathtt{nat} \mid \mathtt{y} > 0\} \Rightarrow \{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} \neq 0\}\rangle^{\ell}\ \underline{1}) \rightsquigarrow$

**篁 c　箴**

div 5 (succ 0)

**篁　箴**

$\mathtt{div}\ \mathtt{5}$
$\quad (\langle\{\mathtt{y} : \mathtt{nat} \mid \mathtt{y} > \mathtt{0}\} \Rightarrow \{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} \neq \mathtt{0}\}\rangle^{\ell}\ (\mathtt{succ}\ \mathtt{0})) \rightsquigarrow$

$\mathtt{div}\ \mathtt{5}$
$\quad (\langle\{\mathtt{y} : \mathtt{nat} \mid \mathtt{y} > \mathtt{0}\} \Rightarrow \{\mathtt{x} : \mathtt{nat} \mid \mathtt{x} \neq \mathtt{0}\}\rangle^{\ell}\ \mathtt{1}) \rightsquigarrow$

$\mathtt{div}\ \mathtt{5}\ (\mathtt{if}\ (\mathtt{1} \neq \mathtt{0})\ \mathtt{then}\ \mathtt{1}\ \mathtt{else}\ \Uparrow \ell) \rightsquigarrow^{*}$

**篁 c　箴**

$\mathtt{div}\ \mathtt{5}\ (\mathtt{succ}\ \mathtt{0})$

**篁　箴**

$\texttt{div 5}$
$\quad (\langle\{y : \texttt{nat} \mid y > 0\} \Rightarrow \{x : \texttt{nat} \mid x \neq 0\}\rangle^{\ell}\ \texttt{(succ 0)}) \rightsquigarrow$

$\texttt{div 5}$
$\quad (\langle\{y : \texttt{nat} \mid y > 0\} \Rightarrow \{x : \texttt{nat} \mid x \neq 0\}\rangle^{\ell}\ \texttt{1}) \rightsquigarrow$

$\texttt{div 5 (if (1} \neq \texttt{0) then 1 else} \Uparrow \ell) \rightsquigarrow^{*}$

$\texttt{div 5 1} \rightsquigarrow$

**篁c　箴**

$\texttt{div 5 (succ 0)}$

**篁 箴**

$\texttt{div 5}$
$\quad (\langle\{\mathrm{y} : \mathtt{nat} \mid \mathrm{y} > 0\} \Rightarrow \{\mathrm{x} : \mathtt{nat} \mid \mathrm{x} \neq 0\}\rangle^{\ell}\ \texttt{(succ 0))} \rightsquigarrow$

$\texttt{div 5}$
$\quad (\langle\{\mathrm{y} : \mathtt{nat} \mid \mathrm{y} > 0\} \Rightarrow \{\mathrm{x} : \mathtt{nat} \mid \mathrm{x} \neq 0\}\rangle^{\ell}\ \texttt{1)} \rightsquigarrow$

$\texttt{div 5 (if (1} \neq \texttt{0) then 1 else} \Uparrow \ell) \rightsquigarrow^{*}$

$\texttt{div 5 1} \rightsquigarrow \texttt{5}$

**篁 c 箴**

$\texttt{div 5 (succ 0)}$

**篁　箴**

$\texttt{div 5}$
$\quad (\langle\{\mathrm{y} : \mathtt{nat} \mid \mathrm{y} > 0\} \Rightarrow \{\mathrm{x} : \mathtt{nat} \mid \mathrm{x} \neq 0\}\rangle^{\ell}\ \texttt{(succ 0))} \rightsquigarrow$

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$\texttt{div 5 (if (1} \neq \texttt{0) then 1 else} \Uparrow \ell\texttt{)} \rightsquigarrow^{*}$

$\texttt{div 5 1} \rightsquigarrow \texttt{5}$

**篁 c　箴**

$\texttt{div 5 (succ 0)}$

**篁　箴**

$\texttt{div 5}$
$\quad (\langle \{\mathrm{y} : \texttt{nat} \mid \mathrm{y} > 0\} \Rightarrow \{\mathrm{x} : \texttt{nat} \mid \mathrm{x} \neq 0\} \rangle^{\ell}\ \texttt{(succ 0)}) \rightsquigarrow$

$\texttt{div 5}$
$\quad (\langle \{\mathrm{y} : \texttt{nat} \mid \mathrm{y} > 0\} \Rightarrow \{\mathrm{x} : \texttt{nat} \mid \mathrm{x} \neq 0\} \rangle^{\ell}\ \texttt{1}) \rightsquigarrow$

$\texttt{div 5 (if (1} \neq \texttt{0) then 1 else} \Uparrow \ell\texttt{)} \rightsquigarrow^{*}$

$\texttt{div 5 1} \rightsquigarrow \texttt{5}$

**篁c　箴**

$\texttt{div 5 (succ 0)} \rightsquigarrow \texttt{div 5 1} \rightsquigarrow \texttt{5}$

# c 鴻ゅ祉

- c 鴻　ゅ祉　箴＜　襲帥
- ≪　c 鴻 $\langle T_1 \Rightarrow T_2 \rangle^\ell$　絵
  - $T_1 <: T_2$ 腴や　c 鴻
  - ≫　激鴻
    - ＜睡茫　膣

$$\langle \{y : \mathrm{nat} \mid y > 0\} \Rightarrow \{x : \mathrm{nat} \mid x \neq 0\} \rangle^\ell$$

# 腥吟 覊

**蕁 絅膣荐膊篏膺 $F_H^{fix}$ c ≪ c 鴻ゅ祉 篋∞ 襲 帥 腓冴**

- ≪ c 鴻 膈∽違 識篁膈箴≧荐惹
  違 膈茵 ≫
- 綣怨篁膈箴≧ c ュ 茫≫
  ゃ 鍵膣 臂 ≫

# 茵腥吟 莠

| 荐膊篏膤 | $\langle \mathtt{T}_1 \Rightarrow \mathtt{T}_2 \rangle^{\ell} \simeq \lambda\, \mathtt{x}.\, \mathtt{x}$ |
|---|---|
| $\lambda^H$ [1] | [1] |
| $\mathsf{F}_{\mathsf{H}}$ [2] | [2] |
| $\mathsf{F}_{\mathsf{H}}^{\mathsf{fix}}$ | ✓ |

$\simeq$ 茫≫

✓ 腥吟


[1] Knowles and Flanagan, 2010

[2] Belo, Greenberg, Igarashi and Pierce, 2011

# 茵腥吟 莠

| 荐膊篏膈 | $\langle T_1 \Rightarrow T_2 \rangle^\ell \simeq \lambda x.\, x$ | $\simeq \subseteq \approx$ | $\langle T_1 \Rightarrow T_2 \rangle^\ell \approx \lambda x.\, x$ |
|---|---|---|---|
| $\lambda^H$ [1] | [1] | 荐惹箏 | $\vdash$ |
| $F_H$ [2] | [2] | $\vdash$ | $\vdash$ |
| $F_H^{fix}$ | ✓ | ✓ | ✓ |

- $\simeq$ 茫≫
- ✓ 腥吟
- $\approx$ 膈箴≧


[1] Knowles and Flanagan, 2010
[2] Belo, Greenberg, Igarashi and Pierce, 2011

## Hybrid(static + dynamic) checking 茵荐膊簌膈

$$::= \quad \mathtt{bool} \mid \mathtt{nat} \mid \ldots \mid \alpha \mid \forall\,\alpha.\,\mathtt{T}$$
$$\{\mathtt{x} : \mathtt{T} \mid \mathtt{e}\} \mid \mathtt{x} : \mathtt{T}_1 \rightarrow \; \mathtt{T}_2$$

$$::= \quad \mathtt{x} \mid \mathtt{k} \mid \mathtt{v}_1\ \mathtt{v}_2 \mid \Lambda\alpha.\ \mathtt{v} \mid \mathtt{v}\ \mathtt{T} \mid \ldots \mid$$
$$\lambda\,\mathtt{f}(\mathtt{x} : \mathtt{T}).\mathtt{e} \mid \quad (\text{紹育ᔕ})$$
$$\langle \mathtt{T}_1 \Rightarrow \mathtt{T}_2 \rangle^{\ell} \quad (\quad \text{c 鴻})$$

$\{x : T \mid e\}$ 膀(泣)(refinement type)

- e 羣 T
  - $e[v/x] \leadsto^* \texttt{true}$ v
  - e 遵ゆ 違

$x : T_1 \to T_2$ 箴紞ゝ医

- 祉ゆ綣違 絖

$$x : \mathtt{nat} \to y : \mathtt{nat} \to \{z : \mathtt{nat} \mid z > x \,\&\&\, z > y\}$$

$\langle T_1 \Rightarrow T_2 \rangle^{\ell} : T_1 \rightarrow T_2$

- **絎茵鑷祉絎**
  - $T_1$ や $T_2$ 鑷
  - **桼掩遺桼榊**
  - $\ell$ **桼 榊贋** ≪

$$\langle \mathtt{nat} \Rightarrow \{x : \mathtt{nat} \mid x \neq 0\} \rangle^{\ell}\ 2 \rightsquigarrow^{*} 2$$
$$\langle \mathtt{nat} \Rightarrow \{x : \mathtt{nat} \mid x \neq 0\} \rangle^{\ell}\ 0 \rightsquigarrow^{*} \Uparrow \ell$$

# 膈箴≧

$e \approx e'$

- e e′ 算 x
  - : 腥眼 違
    箴: $(\lambda\, x : \mathtt{nat}.x + 1)$ []
  - ュ c ⊢
- 贋ョず 育
  - 聋≦ ず羈 ≫ [1] ヤ

[1] Plotkin, 1973

# 絶怨篁膈蔵≧

$\Gamma \vdash e \approx e' : T$

- $e'$ 篁
  - $\langle T_1 \Rightarrow T_2 \rangle^{\ell} \approx \lambda x.\, x$ 腓汧
  - ≪ c 鴻 膈∽違 篁

| | $\langle T_1 \Rightarrow T_2 \rangle^{\ell}$ | $\lambda x : T_1 . x$ | $\lambda x : T_2 . x$ |
|---|---|---|---|
| | $T_1 \rightarrow T_2$ | $T_1 \rightarrow T_1$ | $T_2 \rightarrow T_2$ |

- $e$ $T$ 蚊綵
- 篁荐茯 c $e$ 鐚 $e'$

# 綣怨篁膈箴≧ c 臂

**絎臂 (綣怨篁膈箴≧)**

$\Gamma \vdash e \approx e' : T$ >散羣紊 c

- $\Gamma \vdash e : T$
- $\Gamma = \emptyset$ $e \Downarrow e'$ (苟恰謙 純 膈)
- ゆ: 絨 э Щ э (篁) 絲丞 0
- 篋 (*compatibility*)
- 臀 (*substitutivity*)

# 篋 c 舟 c

- 篋 (compatibility)

$$\frac{\Gamma \vdash v_1 \approx v_1' : (x : T_1 \rightarrow T_2) \qquad \Gamma \vdash v_2 \approx v_2' : T_1}{\Gamma \vdash v_1\ v_2 \approx v_1'\ v_2' : T_2[{\color{red}v_2}/x]}$$

- 臀 (substitutivity)

$$\frac{\Gamma, x : T', \Gamma' \vdash e \approx e' : T \qquad \Gamma \vdash v \approx v' : T'}{\Gamma, \Gamma'[v/x] \vdash e[v/x] \approx e'[v'/x] : T[{\color{red}v}/x]}$$

# 荀恰謙　純　　箴≧

**絎臂 (荀恰謙　純　　箴≧)**

$e \Downarrow e'$ *iff*

- e 罩 *iff* $e'$ 罩
- e　$\Uparrow \ell$ 榊 *iff* $e'$　$\Uparrow \ell$ 榊
- $e'$　*stuck*

$\Gamma \vdash e \simeq e' : T$

- T　　や　鍵膣　臂　≫
- e'　　　篁
    - e　　T 蚊綵
- 絎臂　　　⊤⊤ [1] 綽

[1] Pitts, 2000

1. 堺 や や ≫絎臂

bool: {(true,true), (false,false)}

nat: {(0,0),(1,1),(2,2),...}

# 茫≫絎臂 (TT )

1. 塀 や や ≫絎臂
2. 篏 $\mathcal{ER}$ 塀 や ≫絎臂

bool : {(true, not false),(true && true, true) ...}

nat: {(1+1,2),(0∗3,0+0),...}

ら≫ $\xrightarrow{\text{篏}\ \mathcal{ER}}$ ≫

# 茫≫絎臂 (TT)

1. 堺 や や ≫絎臂
2. 篏 $\mathcal{ER}$ 堺 や ≫絎臂
3. 茲 や や ≫絎臂

$$\mathsf{nat} \rightarrow \mathsf{nat} : \{(\mathsf{succ}, \lambda x. x + 1), \ldots\}$$

# 茫≫絎臂 (TT )

1. 堺 や や ≫絎臂
2. 篏$\mathcal{ER}$ 堺 や ≫絎臂
3. 茲 や や ≫絎臂
4. 篏$\mathcal{ER}$ 茲 や ≫絎臂

# 茫≫絎臂 (TT )

1. 堺 や や ≫絎臂
2. 篏 $\mathcal{ER}$ 堺 や ≫絎臂
3. 茲 や や ≫絎臂
4. 篏 $\mathcal{ER}$ 茲 や ≫絎臂

# ≫ <若 $\theta$ 鍐 $\delta$

- 膀 $\{x : T \mid e\}$ 膛 e 篁
  - e 荅箴<
  - 膀 ら≫ 膛羣 絎

$$v_1 \simeq v_2 : \{x : T \mid e\}$$

$$e[v_1/x] \rightsquigarrow^* \mathtt{true} \quad e[v_2/x] \rightsquigarrow^* \mathtt{true}$$

- $\theta = \{\alpha \mapsto r, T_1, T_2\}$
  - $\theta_i = \{\alpha \mapsto T_i\}$
- $\delta = \{x \mapsto v_1, v_2\}$
  - $\delta_i = \{x \mapsto v_i\}$
  - $v_1$ 鍐 $v_2$ ら≫ 障

# 堺　　　≫

堺　bool

**紵臂 (ら≫ $\mathtt{Id}_{\mathtt{bool}}$)**

$$\mathtt{Id}_{\mathtt{bool}} = \{(\mathtt{true}, \mathtt{true}), (\mathtt{false}, \mathtt{false})\}$$

**紵臂 (≫ $\mathtt{bool}(\theta, \delta)$)**

$$\mathtt{bool}(\theta, \delta) = \mathtt{Id}_{\mathtt{bool}}{}^{\mathcal{ER}}$$

nat　　　や　罕

# 膀 ≫

膀 $\{x : T \mid e\}$

**絎臂 (ら≫ $\{x : r \mid e, e'\}$)**

$(v, v') \in \{x : r \mid e, e'\}$ *iff* $(v, v') \in r$

$e[v/x] \rightsquigarrow^* \texttt{true}$ $e'[v'/x] \rightsquigarrow^* \texttt{true}$

**絎臂 (≫ $\{x : T \mid e\}(\theta, \delta)$)**

$$\{x : T \mid e\}(\theta, \delta) = \{x : T(\theta, \delta) \mid \theta_1\delta_1(e), \theta_2\delta_2(e)\}^{\mathcal{ER}}$$

$x \notin \mathrm{dom}(\delta)$

# 箴統∽医 ≫

箴統∽医 $x : T_1 \to T_2$

**絎臂 (ら≫ $\text{fun } x : r.\ \boldsymbol{\lambda}(v_2, v_2').R(v_2, v_2')$)**

$(v_1, v_1') \in (\text{fun } x : r.\ \boldsymbol{\lambda}(v_2, v_2').R(v_2, v_2'))$ *iff*
$\quad \forall (v_2, v_2') \in r.\ (v_1\ v_2, v_1'\ v_2') \in R(v_2, v_2')$

**絎臂 (≫ $x : T_1 \to T_2(\theta, \delta)$)**

$$x : T_1 \to T_2(\theta, \delta) = (\text{fun } x : T_1(\theta, \delta).\ \boldsymbol{\lambda}(v, v').T_2(\theta, \delta\{x \mapsto v, v'\}))^{\mathcal{ER}}$$

# 茫≫ 臂 ч蟹

**絎臂 ( ≪茫≫ $\Gamma \vdash e \simeq e' : T$)**

$\forall \theta, \delta.\ \Gamma \vdash \theta; \delta$

- $\Gamma \vdash e : T$
- $(\theta_1\delta_1(e), \theta_2\delta_2(e')) \in T(\theta, \delta)$

**絎 (ュ )**

$\Gamma \vdash e \simeq e' : T \qquad \Gamma \vdash e \approx e' : T$

≫ [1] 膣眼 ≦宍

**絎**

$\Gamma \vdash T_1 <: T_2$

$\Gamma \vdash \langle T_1 \Rightarrow T_2 \rangle^{\ell} \simeq (\lambda\, x : T_1 . x) : T_1 \to T_2$

**膺 (≪ c 鴻ゆ)**

$\Gamma \vdash T_1 <: T_2$

$\Gamma \vdash \langle T_1 \Rightarrow T_2 \rangle^{\ell} \approx (\lambda\, x : T_1 . x) : T_1 \to T_2$

[1] Knowles and Flanagan, 2010

# 障 縮 蕁

- 荐膊篏膪 $F_H^{fix}$ や
  - 綣怨篁膈箴≧ c ュ ≫筝
  - ≪ c 鴻 ゅ祉 荅箴∞紊 腓冴
- 篁縮 蕁
  - 茫≫

| 荐膊篏膪 | $\langle T_1 \Rightarrow T_2\rangle^{\ell} \simeq \lambda x.\, x$ | $\simeq \subseteq \approx$ | $\langle T_1 \Rightarrow T_2\rangle^{\ell} \approx \lambda x.\, x$ |
|---|---|---|---|
| $\lambda^H$ | Knowles et al. | 荐惹 | ⊢ |
| $F_H$ | Belo et al. | ⊢ | ⊢ |
| $F_H^{fix}$ | ✓ | ✓ | ✓ |